The Endogeneity of the Exchange Rate as a Determinant of FDI: A Model of Money, Entry, and Multinational Firms

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Abstract

Existing theoretical and empirical studies draw conflicting conclusions as to whether exchange rate volatility will encourage or suppress FDI, largely because they divorce exchange rate fluctuations from economic conditions within the host country. This paper argues that the exchange rate and projected sales in the host country are jointly determined by underlying macroeconomic variables, suggesting that regressions of FDI flows on both exchange rate levels and volatility are subject to bias. The results hinge on the interaction of macroeconomic uncertainty, a sunk cost, and heterogeneous productivity across firms. It is the first study to depart from the representative-firm framework in an analysis of direct investment behavior with money. The results demonstrate that how a multinational firm responds to increases in exchange rate volatility will differ depending on whether the volatility arises from shocks in the firm's native or host country.

JEL Classifications: F1, F2, F4
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1 Introduction

The share of foreign direct investment (FDI) in total net resource flows to developing countries more than doubled during the 1990s, reaching 82 percent in 2001 (IMF 2002). While the developing world saw flows of portfolio investment and external sources of private credit evaporate during and after the East Asian crisis in 1997 and 1998, FDI continued in a steady stream. The phenomenon was not isolated, but occurred across both East Asia and Latin America (World Bank 2003). In the face of this startling dichotomy, the questions of how and why FDI responds to exchange rate fluctuations, first raised in the 1980s, have taken on a new relevance to open-economy macroeconomic analysis. Interestingly, the question of how exchange rate variability impacts the investment decisions of multinational firms is largely ignored in the debate over optimum currency regimes. This paper addresses the question by expanding the general equilibrium analysis characteristic of recent optimum currency area theory– where the exchange rate depends on "fundamental" variables that may also impact local demand[1]– to encompass the entry behavior of the multinational firm.

Although empirical and partial-equilibrium analyses suggest that exchange rate uncertainty may be important in a firm's decision to engage in production activity overseas, the literature incorporating multinational firms into models of the global economy has remained separate from studies of exchange rate behavior and policy. Previous studies of exchange rate variability and multinational firms have often treated the exchange rate as exogenous.[2] The model presented here combines the concept of an exchange rate driven by macroeconomic variables with a sunk cost, which motivates firms' sensitivity to uncertainty in the trade and industrial organization literature. The results indicate that while macroeconomic volatility in the MNE's native country and the country hosting its direct investment venture *both* increase exchange rate volatility, they can have quite different effects on flows of FDI.

In addition, the model here is unique in that it departs from the representative-firm framework to look at the effect of volatility on entry. The introduction of heterogeneous productivity levels across firms, based on Melitz (2003), explains why smaller, less productive firms might be deterred from investing overseas by uncertain macroeconomic conditions, while larger, more productive firms are not. A comparison of the zero-cutoff profit conditions for domestic and foreign entrants also allows the decomposition of the impact of monetary volatility into factors affecting all firms and those affecting only entering foreign firms through their exposure to exchange rate fluctuations. The analysis presented in Section 4 illustrates the two effects and provides a theoretical explanation

[1] See Tamim Bayoumi's (1994) study for the first formalization of OCA theory in a general equilibrium framework.

[2] Notable exceptions are Goldberg and Kolstad (1995), who conjecture that the effect of exchange rate volatility on FDI should depend on the correlation of exchange rate shocks with demand shocks, and Aizenman (1992) and Devereux and Engel (2001), which are the first models incorporating the MNE into a general equilibrium framework with money, endogenous exchange rates, and a representative firm.

for the observation by Hausmann and Fernandez-Arias (2000) that FDI appears to be cushioned from some types of macroeconomic risk that have been shown to curtail other types of private investment. It also reconciles the conflicting estimates of the direction of the effect of exchange rate volatility on flows of FDI evident in empirical studies based on partial equilibrium models.

The argument presented in this paper rests on two premises. First, it assumes that there is a repeated sunk cost involved in production at home and overseas– some fixed overhead cost such as legal retainers, rental and maintenance contracts, or a property tax that is paid, negotiated, or legislated in advance. In this respect, the model draws on the option value literature sparked by Pindyck (1988), Dixit and Pindyck (1994), and Campa (1993), as well as on trade models incorporating multinational firms with plant-level fixed costs (Horstmann and Markusen 1992 and Brainard 1997) and sunk costs, which are defined here as fixed costs that must be paid before the realization of a random shock (Grossman and Razin 1985 and Helpman, Melitz, and Yeaple 2003). Because the sunk cost is paid or negotiated in one period under a given exchange rate, but revenues are earned and repatriated at a later date, firms care about fluctuations in the value of the host-country currency. Firms are not allowed to hedge against fluctuations in the exchange rate in this model.

Second, it is assumed that there are common macroeconomic forces that influence both the exchange rate and the volume of sales by overseas branches. These forces could involve productivity growth or any number of unobservable variables governing the international asset market. However, this study focuses on fluctuations in the growth rate of the money supply as the mechanism influencing both realizations of the exchange rate and, due to sticky prices, the demand for consumption goods in the host country. The exchange rate is a function of the ratio of the home (host-country) and foreign (native-country) money supply. It covaries negatively with the host country's demand for goods, as a positive shock to the home money supply weakens the value of the home currency but simultaneously increases real income—and therefore sales by both domestically owned firms and multinationals operating in the home market. Conversely, a contractionary monetary shock in the host country may generate a more favorable exchange rate at which to convert profits, but will depress local sales. In comparison, a contractionary monetary shock arising in the MNE's native country can adversely affect the value of the home currency with no mitigating influence (or, in the case of complete markets, an exacerbating influence) on sales overseas.

Thus, there are two rigidities driving the results of the model. The real rigidity, a sunk cost, motivates firms' sensitivity to uncertainty. The nominal rigidity, sticky prices, causes the uncertainty generated by monetary shocks to influence both the exchange rate and consumer demand in the host market. Because the exchange rate and the demand for goods are impacted differently by monetary shocks, depending on their origin, the analysis below indicates that studies regressing

FDI flows on measures of exchange rate volatility may be subject to bias. Considering FDI and exchange rates as variables jointly determined by underlying macroeconomic factors provides an explanation of why the empirical literature analyzing whether exchange rate variability encourages or deters investment by multinational firms remains inconclusive.

The remainder of the text is organized as follows: First, insights from existing theoretical and empirical studies which guided the construction of this model are discussed in Sections 1.1 and 1.2. Sections 2.1 and 2.2 describe the consumer's optimization problem and relevant first-order conditions. Section 2.3 defines an expression for expected discounted profits and describes the firm's pricing behavior under uncertainty. Section 2.4 explains the calculation of aggregate productivity and the price level and 2.5 explains how aggregate productivity is related to the "threshold" productivity level, or the labor productivity of the least productive entrant. Part 3 presents the key equilibrium conditions governing investment behavior. A special section, 3.3, is devoted to discussing issues of geographic preference and asset-market structure. An analysis of the results is presented in Part 4, where the decision criteria of foreign investors considering a direct investment venture is decomposed into factors affecting all firms operating in the host market and factors rooted in exchange-rate risk, which affect only entrants from overseas. It evaluates the net impact of monetary policy variables on entry by domestic and foreign-owned firms, as well as implications for aggregate prices and consumption in the host market. Part 5 concludes the paper with a discussion of the results and possibilities for future research.

2 The Theoretical Debate

Existing partial equilibrium models provide important insight into the mechanics of the MNE's decision-making behavior, but treat exchange rate fluctuations as exogenous, isolating them from macroeconomic shocks that simultaneously affect demand. Consequently, theoretical arguments based on these models are divided as to whether exchange rate uncertainty will increase or decrease FDI. Authors proposing that exchange rate variations could promote investment abroad assert the long-standing result in trade theory that cross-border investment is a substitute for trade when tariffs or other barriers prevent the free flow of goods (Goldberg and Kolstad 1995, Cushman 1985 and 1988). Mundell (1957) provides the first mathematical proof of this result and attributes the idea behind it to Ohlin (1935), Iverson (1935), and Meade (1955). Numerous studies provide evidence that exchange rate uncertainty may function as a *de facto* trade barrier, implying by default that it should increase FDI.[3] A related position espouses the "production flexibility" approach–

[3]Examples of such studies include Cushman (1983) and Dell'Ariccia (1999). Cote (1994) and Barkoulas, Baum, and Caglayan (2002) discuss the conflict that exists within the sizeable literature investigating exchange-rate volatility and trade.

that volatility increases the value of having a plant in both countries, enabling an MNE to decide at any time either to export from home or to produce in its foreign facility, depending on where conditions are most favorable (Sung and Lapan 2000). Assuming that exchange rate fluctuations are exogenous, multinational firms can take advantage of them by shifting production to the countries where the value of the local currency makes input costs look cheapest, *ceteris paribus*.[4] In earlier work, Itagaki (1981) develops a financial flexibility argument. He posits that an increase in exchange rate risk may incite a firm to invest abroad as a way of hedging against a short position in its balance sheet. A depreciation of the firm's home currency might reduce the value of domestic assets relative to foreign liabilities, but would simultaneously increase the value of assets and revenue streams for its affiliates in foreign countries.

However, theoretical evidence also exists indicating that exchange rate uncertainty may instead suppress FDI. These arguments assert that unpredictable fluctuations in the exchange rate introduce added uncertainty into both the production costs and future revenues of overseas operations, deterring potential investors. Several studies (Rivoli and Salorio 1996 and Campa 1993), rooted in the work of Pindyck (1988) and Dixit and Pindyck (1994), declare that currency volatility deters the entry of multinational firms by increasing the "option value" associated with waiting before incurring the sunk costs necessary to produce overseas. They consider that a firm effectively holds an option to invest overseas in any given period. A fixed cost paid in advance (sunk) acts as an exercise price. The return from exercising the option is the expected present discounted value of profits earned from production in the foreign country. Exchange rate risk introduces uncertainty about the size of the return, increasing the value of holding on to the option to wait and motivating the firm to postpone investing until a future period. A salient feature of this literature is that the results hold even for risk-neutral firms. The key engine is the sunk cost. Without it, there would be no cost to producing when the prevailing exchange rate allows positive returns and exiting when it does not, eliminating any value attached to waiting.

Several studies providing results indicating that exchange-rate volatility may discourage FDI are driven by risk-averse firm managers rather than sunk costs. Cushman (1985 and 1988) and Goldberg and Kolstad (1995) specify conditions under which exchange-rate volatility may reduce the certainty-equivalence value of expected profits from overseas operations– a deterrent to prospective investors. They show that a deterrent effect arises as long as demand and the elasticity of technical substitution are of a form such that they completely offset the effect of currency fluctuations on

[4] An emerging literature focuses on the implications of skewness in the distribution of exchange rate shocks, demonstrating that investors will hurry to invest in a country whose currency appears to have suddenly depreciated to a level below its expected value. Such an extreme depreciation reduces the burden of fixed entry costs paid in local currency, resulting in a temporary surge in "firesale" FDI (Chakrabarti and Scholnick 2002, Krugman 1998). This research is inspired by studies such as Froot and Stein (1991) and Blonigen (1997) (and challenged by Stevens (1998)), which analyze FDI and exchange rate levels.

profit remittances. More concretely, exchange rate volatility will deter FDI as long as a depreciation of the host-country currency, which would undercut the value of repatriated profits in terms of the firm's native-country currency, is not met by an offsetting increase in host-country demand or reduction in host-country input costs. Like the literature contending that uncertainty promotes FDI, these studies either overlook any simultaneous effects of underlying variables on demand and the exchange rate, or consider a correlation between the two without explicitly characterizing its relationship to underlying fundamental variables.

There are two studies which incorporate multinational firms within a general equilibrium approach, where the exchange rate is endogenous. They are remarkable in that they are the first to incorporate the multinational firm into an open-economy framework with money. Notwithstanding, it is difficult to interpret them in light of the question, "Does exchange rate volatility deter FDI?" because they incorporate a representative firm, implying that either all firms invest abroad or none do. The first study, Aizenman (1992), juxtaposes the production flexibility approach onto the Dixit-Pindyck conceptualization of the option value.[5] Aizenman builds a framework where an important motivation for FDI is what he terms "diversification"– the ability to shift production across borders quickly as a hedge against country-specific real and monetary shocks. Firm-level increasing returns to scale constitute a second motive, as a second plant overseas can be established with a slightly lower fixed cost than the original plant in the firm's native country. In addition, his model includes a sunk capital-investment cost, a lump sum which firms must pay one period ahead of production when they establish a plant overseas. The option value competes with the diversification and returns-to-scale incentives in the presence of idiosyncratic shocks to each country's money supply and productivity level. Increases in volatility increase the value of diversification, which pushes firms to shift production to the country where it is cheapest, but also discourages investment by increasing the uncertainty surrounding the return on exercising the Dixit-Pindyck financial option to invest abroad. He demonstrates that a floating exchange rate will transmit the effect of these idiosyncratic shocks across national borders, which erodes the ability of firms to diversify risk by shifting production across borders. When shocks are transmitted between countries in this way, the deterent effect of the Dixit-Pindyck option value overrides the production-flexibility and returns-to-scale motives, causing volatility in the money supply and the productivity level that is transmitted through fluctuations in the exchange rate to prevent firms from diversifying by investing abroad. In this sense, the exchange rate volatility associated with a flexible regime can be construed as deterring FDI.

[5] Aizenman explicitly discusses only the value of having the option to produce in either of two plants located in two different countries (production flexibility). He does not discuss the financial option value in the sense of Dixit and Pindyck's work or draw parallels with this idea. However, the sunk investment cost in his model creates precisely the same engine, deterring investment in the face of uncertainty.

The second study, Devereux and Engel (2001), assumes Cobb-Douglas preferences in the consumption of goods produced by home and foreign firms. The model abstracts from trade in goods, so the unit elasticity between home and foreign goods forces the representative home to invest abroad to satisfy foreign consumers' demand for its good. This means that all firms always invest abroad, so it is particularly difficult to assess the impact of the volatility of macroeconomic shocks on firms' willingness to undertake a direct investment venture. In the case where firms price in the currency of the local market they are serving (pricing-to-market), production by all firms, including the affiliates of multinationals, is higher under a flexible exchange rate than a fixed exchange rate. Hence, exchange-rate volatility associated with a flexible regime is loosely linked here with increased production by multinationals.

3 Empirical Evidence

Existing empirical tests are based on the above partial equilibrium models or on gravity models and offer two principal conclusions: (1) it is not clear what relationship exists between exchange-rate uncertainty and FDI and (2) local fixed costs make it more likely that FDI will be discouraged by exchange-rate volatility. With regard to the first issue, both Cushman (1985 and 1988) and Goldberg and Kolstad (1995) find that volatility increases the willingness of U.S. MNEs to locate facilities abroad, in accordance with the early trade theory explaining FDI as a substitute for exports. Zhang (2001) supports their results, finding a positive and significant relationship between exchange rate volatility and FDI flowing into the European Union (EU) from both inside and outside the EU. Nevertheless, there is ample evidence to the contrary. Whereas Sekkat and Galgau (2004) also find a positive link for flows between EU nations, they find that increases in the variance of bilateral exchange rates deter inflows originating outside of the EU. Amuedo-Dorantes and Pozo (2001) report that results may not be robust to the way volatility is measured: there is a positive coefficient associated with volatility of the exchange rate measured as a the standard deviation within a rolling window but a negative coefficient emerges when a GARCH construction is used. Both coefficients are significant. Chakrabarti and Scholnick (2002) find a negative relationship between exchange rate volatility and FDI flows from the U.S. to 20 OECD countries. Using micro-level data, Campa (1993) shows that volatility deters entry by foreign firms contemplating investment in the U.S.

Interestingly, there is consensus on the role of fixed costs and FDI. Several papers offer evidence that the fixed costs prominent in trade and industrial-organization literature examining multinational firms are important in understanding the response of MNEs to exchange rate uncertainty.[6] Campa's (1993) study shows that the negative impact of exchange rate uncertainty on entry by MNEs is more probable and more profound when sunk costs are large. Tomlin (2000) demonstrates that local fixed costs, such as advertising expenses, are alone sufficient to quell entry by foreign firms deciding whether to produce in the U.S. The analysis of Sekkat and Galgau (2001) indicates that although the exchange rate volatility-FDI link is positive overall for intra-EU flows, it is negative for the rubber, petroleum, chemical, and plastic products industries– manufacturing industries with relatively high plant-level (local) fixed costs. Alaba (2003) similarly finds that whereas the relationship is positive for the agricultural sector in Nigeria, it is negative for manufacturing. Campa (1993) explains this phenomenon using the options theory reasoning: in the *absence* of sunk costs, a firm would simply produce overseas in any period when conditions were favorable, "and volatility would have no effect on the entry decision (p.619)."

Finally, Goldberg and Kolstad (1995) present empirical evidence explicitly calling into question the practice of considering exchange rate volatility and shocks to demand in the host-country market as separate entities. They emphasize that for the majority of the sample used in their analysis, a depreciation of host-country currency is associated with a simultaneous increase in the host-country's demand for goods (Goldberg and Kolstad 1995, p.866). They further show that the share of capacity firms choose to locate abroad may be affected by covariance between host country demand and exchange rate movements, but provide mixed evidence as to the direction of the relationship and its robustness across countries. The result invites the construction of a general equilibrium framework tying demand and the bilateral exchange rate to common underlying fundamental variables, depicting the multinational firm's response to the net effect that macroeconomic shocks exert on both the exchange rate and sales abroad. This paper is the first to introduce FDI into a new open-economy macroeconomic model– where exchange rates and local demand are jointly determined– while preserving the fixed-cost component emphasized in analyses of direct investment within the trade and industrial-organization literature and introducing the heterogeneity which governs entry dynamics in recent trade models.[7]

4 The Model

The model below capitalizes on the insights of partial equilibrium theoretical examinations of MNE behavior and existing empirical evidence, while exploiting the capacity of a general equilibrium framework to connect both demand and the exchange rate to fluctuations in a common underlying variable– money. It builds on the conceptualization of MNEs put forth by Devereux and Engel

[6] See Russ (2004) for a survey of FDI and fixed costs in trade and I-O theory.

[7] For a survey and empirical testing of the implications of heterogeneity in trade theory, see Bernard, Jensen, and Schott (2003).

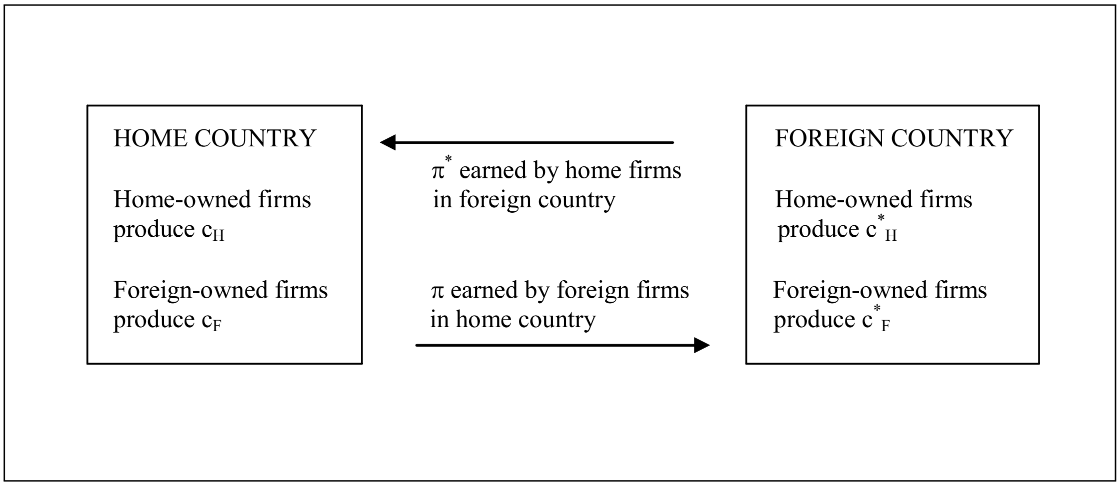

Figure 1: Global Map

(2001), the first study that has incorporated multinational firms into an Obstfeld-Rogoff-type general equilibrium model with money. In their model with two countries (Home and Foreign), there is no trade in goods– all goods are produced in the country where they are consumed. Only the profits of multinational firms cross national borders, as in Figure 1. The model here incorporates a local (i.e., plant-level) sunk cost to motivate the sensitivity of direct investment activity to exchange-rate volatility observed in the data. It also adds heterogeneity across firms to explain why exchange rate uncertainty combines with the sunk cost to deter entry by some firms– but not all– into the overseas market.

To this end, this paper superimposes Devereux and Engel's MNE onto a framework originally developed by Melitz (2003) to explain the decision of firms to export. Melitz's analysis does not include money, but introduces heterogeneity among firms through a random productivity parameter. It accounts for the entry and exit of firms within the industry, while allowing positive profits for the most productive firms. This approach has been used by Helpman, Melitz, and Yeaple (2003) to explain why some firms export and others invest abroad, also in an economy without money. The two-country model here involves money and modifies the optimizing behavior of consumers and producers in Melitz's study by introducing risk aversion.[8]

[8]The study here does not assume *ex ante* that wages are equal across countries. However, it is confined to a two-country world, rather than Melitz's more generalized n-country economy. To simplify the analysis further, it is assumed that firms costlessly find out their productivity parameter, rather than paying a fee before drawing it.

4.1 The Consumer's Problem

In the open-economy model, the representative consumer in the Home country maximizes lifetime utility subject to an intertemporal budget constraint in a setting of complete international asset markets:

$$\max_{\{C_t, B_{t+1}, L_t, M_t\}} E_t \left[\sum_{t=0}^{\infty} \beta^t U_t(C_t, \frac{M_t}{P_t}, L_t) \right]$$

$$s.t.\ P_t C_t + M_t + \sum_{z^{t+1}|z^t} q(z^{t+1}|z^t) B(z^{t+1}) = W_t L_t + \pi_t + M_{t-1} + B_t$$

where $0 < \beta < 1$ and $q(z^{t+1}|z^t)$ is the price at time t of the bond $B(z^{t+1})$, which is denominated in Home currency and has a payoff of one unit of home currency given that one of a set of possible states (z) of the macroeconomy is realized at time at the end of time $t+1$.[9] Utility is a function of aggregate consumption, C, and labor, L,

$$U_t = \frac{1}{1-\rho} C_t^{1-\rho} + \chi \ln \left(\frac{M_t}{P_t} \right) - \kappa L_t.$$

Aggregate consumption is an index reflecting preferences with constant elasticity of substitution (CES) across the set of all goods that could potentially be produced by both Home- and Foreign-owned firms,[10]

$$C_t = \left[\int_0^1 c_H(i,t)^{\frac{\mu-1}{\mu}} di + \int_1^2 c_F(i,t)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}}.$$

Appendix A details the derivation of the aggregate price index,

$$P_t = \left[\int_0^1 p_H(i,t)^{1-\mu} di + \int_1^2 p_F(i,t)^{1-\mu} di \right]^{\frac{1}{1-\mu}}. \quad (1)$$

The demand curves for individual Home and Foreign goods are given by

$$c_H(i,t) = \left(\frac{p_H(i,t)}{P_t} \right)^{-\mu} C_t \qquad c_F(i,t) = \left(\frac{p_F(i,t)}{P_t} \right)^{-\mu} C_t. \quad (2)$$

Multiplying $c_j(i,t)$ by $p_j(i,t)$ for $j = H, F$, an expression for the total expenditure on a particular

[9] $\sum_{z^t|z^{t-1}} B(z^t)$ represents a complete set of state-contingent bonds denominated in the Home currency. The probability of particular states are not explicitly represented here to simplify the exposition. They can be seen in Chari, Kehoe, and McGrattan (2002).

[10] H stands for variables corresponding to Home-owned firms and F for variables corresponding to Foreign-owned firms.

good, $r_j(i,t)$, can be derived

$$r_H(i,t) = p_H(i,t)c_H(i,t) = \left(\frac{p_H(i,t)}{P_t}\right)^{1-\mu} R_t \quad (3)$$

$$r_F(i,t) = p_F(i,t)c_F(i,t) = \left(\frac{p_F(i,t)}{P_t}\right)^{1-\mu} R_t, \quad (4)$$

where $R_t = P_t C_t$, the total consumer expenditure in the Home country. Since expenditures are the flip side of revenues in a general equilibrium framework, total expenditures on a particular good also equal total revenues for the firm which produces it. Analogous expressions apply in the Foreign country.[11]

4.2 First-Order Conditions and the Exchange Rate

First-order conditions, as in Devereux and Engel (2001),[12] yield the wage relation,

$$W_t = \kappa P_t C_t^{\rho}; \quad (5)$$

a money-demand equation,

$$\frac{M_t}{P_t} = \frac{\chi}{1 - E_t[d_{t+1}]} C_t^{\rho}; \quad (6)$$

where $d_{t+1} = \beta \frac{P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}}$, the consumption-based nominal interest rate; and a bond-pricing equation,

$$q(z^{t+1}|z^t) = \beta \frac{P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}}. \quad (7)$$

Rearranging the money demand equation yields an expression for consumption as a function of real money balances and the consumption-based nominal interest rate,

$$C_t^{\rho} = \frac{M_t}{P_t} \left(\frac{1 - E_t[d_{t+1}]}{\chi}\right). \quad (8)$$

It is assumed that the growth rate of the money supply is a lognormally distributed random variable

[11] That is,

$$c_H^*(i,t) = \left(\frac{p_H^*(i,t)}{P_t^*}\right)^{-\mu} C_t^* \qquad c_F^*(i,t) = \left(\frac{p_F^*(i,t)}{P_t^*}\right)^{-\mu} C_t^*$$

$$r_H^*(i,t) = \left(\frac{p_H^*(i,t)}{P_t^*}\right)^{1-\mu} R_t^* \qquad r_F^*(i,t) = \left(\frac{p_F^*(i,t)}{P_t^*}\right)^{1-\mu} R_t^*.$$

[12] First-order conditions are derived explicitly in this paper in Appendix B.

defined by

$$\frac{M_t}{M_{t-1}} = (1+\psi)e^{v_t},$$

where ψ is a constant and ν_t is an i.i.d. random variable with a normal distribution of mean $-\frac{1}{2}\sigma_m^2$ and constant variance σ_m^2.[13] The specification implies that

$$E_t\left[\frac{M_t}{M_{t-1}}\right] = (1+\psi),$$

Obstfeld and Rogoff (1998, p.39) provide an exact solution to (8) in the special case of logarithmic preferences for real balances used here. The solution demonstrates that consumption in a given period is a function only of real money balances and underlying parameters, or

$$C_t^{\rho} = \frac{M_t}{P_t}\left(\frac{1-\beta\theta}{\chi}\right), \tag{9}$$

where $\theta = E_t\left[\frac{M_{t-1}}{M_t}\right] = \frac{e^{\sigma_m^2}}{1+\psi}$, a constant restricted in this analysis so that $\theta < \frac{1}{\beta}$.

Let S_t represent the nominal exchange rate, expressed as units of Home currency per unit of Foreign currency. Then, the bond-pricing equations for the Home- and Foreign-country consumers can be set equal to each other and iterated to show that the real exchange rate in this model is equal to the ratio of the marginal utility of consumption in each country:[14]

$$\frac{S_t P_t^*}{P_t} = \frac{C_t^{*-\rho}}{C_t^{-\rho}}. \tag{10}$$

The expressions for consumption and the real exchange rate in (9) and (10) combine to reveal that the nominal exchange rate in any period is a function of the Home and Foreign money supplies:

$$S_t = \frac{M_t(1-\beta\theta)}{M_t^*(1-\beta\theta^*)}. \tag{11}$$

Uncertainty regarding the exchange rate in this model stems directly from the randomly distributed disturbance in the the growth rate of the money supply in each country. Increased volatility in the disturbances implies greater uncertainty with regard to future levels of the exchange rate.

[13] The results are qualitatively identical whether or not the money-supply growth process is characterized by a mean preserving spread (i.e., whether v_t is distributed $N(-\frac{1}{2}\sigma_m^2, \sigma_m^2)$ or $N(0, \sigma_m^2)$.

[14] See Appendix B.2 for a detailed derivation, as outlined in Chari, Kehoe, and McGrattan (2002).

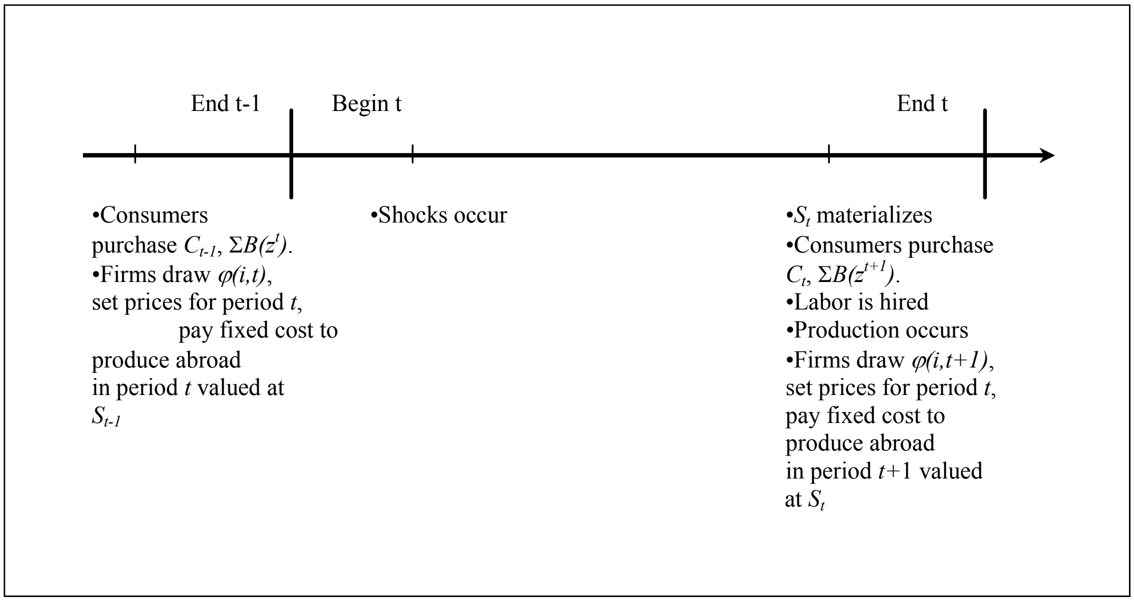

Figure 2: Timeline

4.3 Firms

It is useful at this point to illustrate the timeline of events. Firms each produce a unique good and a have a different productivity index, φ, drawn as an independent, identically distributed random variable at the point in time that the firm decides to enter the industry. The firm learns this parameter and decides whether or not to produce in a given period t at the very end of period $t-1$, as in Figure 2. Production is linear in labor and is characterized by the technology

$$c_H(i,t) = \varphi_H(i,t) l_H(i,t), \tag{12}$$

where $\varphi_H(i,t)$ is the productivity draw of a Home firm i at the beginning of period t. The parameter is known at the time the firm decides whether to produce and what price to charge. The quantity $l_H(i,t)$ is the amount of labor used by Home firm i in its domestic plant in period t. Variables for consumption and production activity in the Foreign country are denoted by an asterisk, so that the identical technology for production by a Home firm abroad is represented by

$$c_H^*(i,t) = \varphi_H(i,t) l_H^*(i,t).$$

The firm seeks to maximize the expected market value of total nominal profits from domestic and overseas plants. Producers anticipate potential fluctuations in demand and wages in the host country as a result of volatility in the host-country money supply. In addition, they consider potential fluctuations in the exchange rate when deciding whether to enter the overseas market, which could occur due to monetary shocks both in the host-country and in its native country (that is, due to shocks to both M and M^*). They therefore place a subjective value on each potential state of the economy using a stochastic discount factor, $d_t = \beta \frac{P_{t-1} C_{t-1}^{\rho}}{P_t C_t^{\rho}}$ for Home firms, and $d_t^* = \beta \frac{P_{t-1}^* C_{t-1}^{*\rho}}{P_t^* C_t^{*\rho}}$ for Foreign firms. The stochastic discount factor is the expected ratio of marginal utility in the present and immediate future, which serves as a measure of how much a shock in period t will impact the well-being of the consumers who own the firm (Cochrane 2001, Chapter 1). As mentioned in Section 2.2, the discount factor, d_t, is also a consumption-based nominal interest rate. In this sense, it represents the opportunity cost of investing in productive activities rather than in bonds at the end of period $t-1$. The results are qualitatively similar whether the stochastic discount factor is used or a constant, $\bar{d}$. This means that, as in the Dixit-Pindyck framework, the results do not depend on whether firm managers are risk-averse or risk-neutral.

The firm's problem is not only to decide whether and how much to produce in its own country, but also whether to undertake production abroad:[15]

$$\max E_{t-1}[d_t \pi_H^T(i,t)],$$

where

$$\pi_H^T(i,t) = E_{t-1}[d_t \pi_H(i,t)] + \max\left\{0, E_{t-1}[d_t \pi_H^*(i,t)]\right\}, \tag{13}$$

$$\pi_H(i,t) = p_H(i,t) c_H(i,t) - W_t l_H(i,t) - P_t f \ ,$$

and

$$\pi_H^*(i,t) = S_t p_H^*(i,t) c_H^*(i,t) - S_t W_t^* l_H^*(i,t) - S_{t-1} P_t^* f_{MNE}^*.$$

Given its knowledge of $\varphi_H(i,t)$ and its expectations of economic conditions in the next period, each Home-owned firm decides whether to pay a fixed overhead cost, $P_t f$, in the Home country and $S_{t-1} P_t f_{MNE}^*$ in the Foreign country, if it chooses to invest overseas, becoming a multinational enterprise. Therefore, to calculate profits from operations abroad, a Home firm takes into account the exchange rate, S_t, at which it will have to pay wages for Foreign workers and repatriate revenues earned overseas, as well as the fixed overhead costs, which it agrees to pay before shocks materialize at the exchange rate S_{t-1}. The cost can be considered "sunk" because it is paid before the firm knows what its revenues will be from sales in the following period. However, it differs from the sunk

[15] Define the expression $E_{t-1}[d_{t-1} \pi_H^T(i,t)]$ as $E[d_{t-1} \pi_H^T(i,t) | \Omega_{t-1}]$, where Ω_{t-1} is the set of all variables observed through the end of period $t-1$ (which includes$\varphi(i,t)$).

cost conceptualized in Melitz (2003), which is an amount paid to find out $\varphi_H(i,t)$. To facilitate tractibility in the open-economy macroeconomic framework, this model assumes that firms costlessly draw $\varphi_H(i,t)$. As a result, this model abstracts from certain properties of firm dynamics, such as the age of the average producing firm, which are emphasized in recent trade models.

It is useful to emphasize here that this is a model of *horizontal* direct investment, where a firm produces a unique good in multiple countries, but for local consumption in each country, not for trade or assembly somewhere else. The model abstracts from cross-border flows of physical capital. Cross-border capital flows, where imported capital is used in domestic production using domestic technology, are distinct from FDI, where a firm uses a single technology to produce identical goods in multiple countries. (See Russ (2004) for a more detailed discussion and a literature review of studies modeling cross-border capital flows.) FDI in the case here is the payment of some fixed cost $(S_{t-1}P_t^* f_{MNE}^*)$ to gain entry into the local market for a particular period. Examples of such recurring fixed costs include annual property taxes, retainer fees for local accounting and legal firms, or fees involved in maintaining local marketing and distribution networks.

Substituting $\frac{c_H(i,t)}{\varphi_H(i,t)}$ for $l_H(i,t)$ and taking the derivative of $E_{t-1}[d_t \pi_H^T(i)]$ with respect to $c_H(i,t)$, one can calculate the price a firm will set when it decides whether to enter the market:

$$\frac{\partial E_{t-1}[d_t \pi_H^T(i)]}{\partial c(i,t)} : E_{t-1}\left[d_t\left(p_H(i,t) + \left(\frac{\partial p_H(i,t)}{\partial c_H(i,t)}\right) c_H(i,t) - \frac{W_t}{\varphi_H(i,t)}\right)\right] = 0$$

$$E_{t-1}\left[d_t\left(p_H(i,t) + \left(-\frac{1}{\mu}\right) p_H(i,t) - \frac{W_t}{\varphi_H(i,t)}\right)\right] = 0$$

$$p_H(i,t) = \left(\frac{\mu}{\mu-1}\right)\frac{E_{t-1}\left[d_t\left(\frac{W_t}{\varphi_H(i,t)}\right)\right]}{E_{t-1}\left[d_t\right]}.$$

A firm will set a price for its unique good equal to a fixed markup over the expected discounted marginal cost. If firms draw from a continuous distribution of labor productivity levels, the probability that multiple firms draw the same productivity parameter is zero, so that one can identify each firm's pricing and production behavior using only φ and drop the firm subscripts. Assuming that each firm faces the same wage level (W_t) and discount factor (d_t), the firm's pricing rule can be written as a function of φ,

$$p_H(\varphi_H(t)) = \frac{E_{t-1}\left[d_t W_t\right]}{\alpha\varphi_H(t) E_{t-1}\left[d_t\right]}, \tag{14}$$

where α is the inverse of the markup $(\alpha = \frac{\mu-1}{\mu})$. Substituting the wage relation and the formula for consumption, equations (5) and (9), the pricing rule is

$$p_H(\varphi_H(t)) = \frac{\kappa(1-\beta\theta)}{\alpha\chi\varphi_H(t)E_{t-1}\left[\frac{1}{M_t}\right]} \tag{15}$$

For Foreign-owned firms deciding whether to operate in the Home market, the pricing rule reduces to a similar expression,

$$\begin{aligned} p_F(\varphi_F(t)) &= \frac{E_{t-1}\left[d_t^* \frac{W_t}{S_t}\right]}{\alpha\varphi_F(t)E_{t-1}\left[d_t^*\left(\frac{1}{S_t}\right)\right]} \\ &= \frac{\kappa(1-\beta\theta)}{\alpha\chi\varphi_F(t)E_{t-1}\left[\frac{1}{M_t}\right]} \end{aligned} \tag{16}$$

Since the markup, the expected discounted wage, and the aggregate price level are the same for all firms, the ratio of output for any two firms a and b operating in the Home economy in period t is a function only of the ratio of $\varphi^a(t)$ to $\varphi^b(t)$:[16]

$$\frac{c(\varphi^a(t))}{c(\varphi^b(t))} = \frac{p(\varphi^a(t))^{-\mu}P_t^{\mu}C_t}{p(\varphi^b(t))^{-\mu}P_t^{\mu}C_t} = \frac{\left(\frac{\kappa(1-\beta\theta)}{\alpha\chi\varphi^a(t)E_{t-1}\left[\frac{1}{M_t}\right]}\right)^{-\mu}}{\left(\frac{\kappa(1-\beta\theta)}{\alpha\chi\varphi^b(t)E_{t-1}\left[\frac{1}{M_t}\right]}\right)^{-\mu}} = \left(\frac{\varphi^a(t)}{\varphi^b(t)}\right)^{\mu}. \tag{17}$$

Similarly, the revenues of any two firms are also a function of the ratio of their productivity levels,

$$\frac{r(\varphi^a(t))}{r(\varphi^b(t))} = \frac{p(\varphi^a(t))^{1-\mu}P_t^{\mu-1}R_t}{p(\varphi^b(t))^{1-\mu}P_t^{\mu-1}R_t} = \frac{\left(\frac{\kappa(1-\beta\theta)}{\alpha\chi\varphi^a(t)E_{t-1}\left[\frac{1}{M_t}\right]}\right)^{1-\mu}}{\left(\frac{\kappa(1-\beta\theta)}{\alpha\chi\varphi^b(t)E_{t-1}\left[\frac{1}{M_t}\right]}\right)^{1-\mu}} = \left(\frac{\varphi^a(t)}{\varphi^b(t)}\right)^{\mu-1}. \tag{18}$$

4.4 Aggregation

There is a continuum of prospective entrants owned by the Home country over the [0,1] interval and of prospective entrants owned by Foreign agents over the interval (1,2]. All prospective entrants costlessly draw a productivity parameter. However, because the fixed costs act as barriers to entry, only a certain fraction of prospective entrants will actually enter the Home market and produce

[16] This key characteristic of the model is derived by substituting the pricing rules (15 and 16) into the demand equations (2).

goods for consumption. Define n_H as the proportion of Home-owned firms entering the Home economy and n_F as the proportion of Foreign-owned firms ($n_H, n_F \leq 1$). Then, there will be a continuum of goods produced by Home firms over $[0, n_H]$ and by Foreign firms over $(1, 1+n_F]$ in the Home market. In equilibrium, there will also be a distribution of productivity levels describing firms that decide to produce– $\eta_H(\varphi)$ for entering Home firms and $\eta_F(\varphi)$ for entering Foreign firms, each with positive support over a subset of $(0, \infty)$. The distribution $\eta_j(\varphi)$ (for $j = H, F$) reflects the probability that a firm has drawn a particular productivity level, given that the firm chose to enter the market. It is assumed that all firms draw from an identical underlying distribution of available technologies, so that $\eta_H(\varphi)$ is the same for all Home firms and $\eta_F(\varphi)$ is the same for all Foreign firms.

Define $\bar{p}_H(t)^{1-\mu}$ as the expected contribution to the overall price level of a Home good chosen at random among Home firms in the economy. Then, because prices differ only according to each firm's level of productivity, $\bar{p}_H(t)^{1-\mu}$ is computed by using the distribution of entrants' productivity levels to find the average contribution that the price of each available good makes to the aggregate price level:

$$\bar{p}_H(t)^{1-\mu} = \int_0^\infty p_H(\varphi_H(t))^{1-\mu} \eta_H(\varphi(t))\, d\varphi.$$

Since the equilibrium distribution is the same for all firms, $\bar{p}_H(t)^{1-\mu}$ is also the same across all Home firms. Similarly, the expected price of a good chosen at random among Foreign firms in the economy is

$$\bar{p}_F(t)^{1-\mu} = \int_0^\infty p_F(\varphi_F(t))^{1-\mu} \eta_F(\varphi(t))\, d\varphi.$$

The aggregate price level can be computed as though all Home-owned firms charged $\bar{p}_H(t)^{1-\mu}$ and all Foreign-owned firms charged $\bar{p}_F(t)^{1-\mu}$, as in Melitz (2002, p.7):

$$\begin{aligned} P_t &= \left[\int_0^{n_H} \bar{p}_H(t)^{1-\mu} di + \int_1^{1+n_F} \bar{p}_F(t)^{1-\mu} di \right]^{\frac{1}{1-\mu}} \\ &= \left[n_H \bar{p}_H(t)^{1-\mu} + n_F \bar{p}_F(t)^{1-\mu} d\varphi \right]^{\frac{1}{1-\mu}}. \end{aligned}$$

Substituting back in the definitions of $\bar{p}_H(t)^{1-\mu}$ and $\bar{p}_F(t)^{1-\mu}$, the price level can be rewritten as

$$P_t = \left[n_H \int_0^\infty p_H(\varphi(t))^{1-\mu} \eta_H(\varphi(t))\, d\varphi + n_F \int_0^\infty p_F(\varphi(t))^{1-\mu} \eta_F(\varphi(t))\, d\varphi \right]^{\frac{1}{1-\mu}}$$

or, using the pricing rules from the firm's maximization problem,

$$P_t = \begin{bmatrix} n_H \int\limits_0^{\infty} \left(\frac{E_{t-1}[d_t W_t]}{\alpha \varphi_H(t) E_{t-1}[d_t]} \right)^{1-\mu} \eta_H \left(\varphi \left(t \right) \right) d\varphi + \\ n_F \int\limits_0^{\infty} \left(\frac{E_{t-1}[d_t^* W_t]}{\alpha \varphi_H(t) E_{t-1}[d_t^*]} \right)^{1-\mu} \eta_F \left(\varphi \left(t \right) \right) d\varphi \end{bmatrix}^{\frac{1}{1-\mu}}.$$

Using (15) and (16) and collecting terms, the expression reduces to

$$P_t = \frac{\kappa(1-\beta\theta)}{\alpha\chi E_{t-1}\left[\frac{1}{M_t}\right]} \left[n_H \int_0^{\infty} \varphi_H(t)^{\mu-1} \eta_H\left(\varphi(t)\right) d\varphi + n_F \int_0^{\infty} \varphi_F(t)^{\mu-1} \eta_F\left(\varphi(t)\right) d\varphi \right]^{\frac{1}{1-\mu}}. \quad (19)$$

To simplify the notation, let $\bar{\varphi}_H(t)$ and $\bar{\varphi}_F(t)$, defined by

$$\begin{aligned} \bar{\varphi}_H(t)^{\mu-1} &= \int_0^{\infty} \varphi_H(t)^{\mu-1} \eta_H\left(\varphi(t)\right) d\varphi \\ \bar{\varphi}_F(t)^{\mu-1} &= \int_0^{\infty} \varphi_F(t)^{\mu-1} \eta_F\left(\varphi(t)\right) d\varphi, \end{aligned} \quad (20)$$

denote the output-weighted average level of productivity of Home and Foreign firms, respectively, operating in the Home economy during period t.[17] Then, the aggregate productivity level for the entire Home economy is given by

$$\bar{\varphi}_t = \left[\frac{n_H}{N} \bar{\varphi}_H(t)^{\mu-1} + \frac{n_F}{N} \bar{\varphi}_F(t)^{\mu-1} \right]^{\frac{1}{\mu-1}}, \quad (21)$$

[17] More precisely, Melitz (2003) points out that $\bar{\varphi}_H(t)$ and $\bar{\varphi}_F(t)$ are expressions of the output-weighted harmonic mean of productivity levels for Home and Foreign firms operating in the Home economy during period t. To see this, note that (for $j = H, F$) the ratio of output for any two firms a and b will be

$$\frac{c_j(\varphi_j^a(t))}{c_j(\varphi_j^b(t))} = \left(\frac{\varphi_j^a(t)}{\varphi_j^b(t)} \right)^{\mu}.$$

Then,

$$\begin{aligned} \bar{\varphi}_j(t)^{\mu-1} &= \int_0^{\infty} \varphi_j(t)^{\mu-1} \eta_j\left(\varphi(t)\right) d\varphi \\ \bar{\varphi}_j(t)^{-1} &= \int_0^{\infty} \varphi_j(t)^{-1} \left(\frac{\varphi_j(t)}{\bar{\varphi}_j(t)} \right)^{\mu} \eta_j\left(\varphi(t)\right) d\varphi \\ &= \int_0^{\infty} \varphi_j(t)^{-1} \left(\frac{c_j(\varphi(t))}{c_j(\bar{\varphi}_j(t))} \right)^{\mu} \eta_j\left(\varphi(t)\right) d\varphi. \end{aligned}$$

where N equals $n_H + n_F$, the composite continuum (a measure of the total variety) of Home and Foreign goods actually produced in the Home country. Using (19) and (21), the aggregate price level can now be expressed as

$$P_t = \frac{\kappa(1-\beta\theta)N^{\frac{1}{1-\mu}}}{\alpha\chi E_{t-1}\left[\frac{1}{M_t}\right]\bar{\varphi}_t}. \tag{22}$$

The aggregate price level is thus a function of the aggregate productivity level, $\bar{\varphi}$.

4.5 Investment Behavior and Productivity

Only firms sufficiently productive to cover their fixed costs will enter and produce in the market, implying that there is some threshold productivity level, $\hat{\varphi}$, below which a firm will not be able to enter the market with the expectation of positive profits. The distribution of successful firms' productivity levels, $\eta_j(\varphi)$, is therefore the probability of drawing a particular φ, given that $\varphi \geq \hat{\varphi}_j$. Let all firms draw from the same (stationary) distribution of productivity levels, with density $g(\varphi)$ and cumulative distribution $G(\varphi)$. Then the probability that a Home-owned firm will draw a particular level of productivity, given that it enters and produces in the Home market, is

$$\eta_H(\varphi) = \eta_H(\varphi_H(t), \hat{\varphi}_H(t)) = \begin{cases} \frac{g(\varphi_H(t))}{1-G(\hat{\varphi}_H(t))} \text{ if } \varphi_H(t) \geq \hat{\varphi}_H(t) \\ 0 \text{ if } \varphi_H(t) < \hat{\varphi}_H(t). \end{cases} \tag{23}$$

This definition of the equilibrium distribution acknowledges that any Home firm which draws $\varphi_H(t) < \hat{\varphi}_H(t)$ exits the market before initiating production. There is a similar distribution for Foreign firms operating in the Home market,

$$\eta_F(\varphi) = \eta_F(\varphi_F(t), \hat{\varphi}_F(t)) = \begin{cases} \frac{g(\varphi_F(t))}{1-G(\hat{\varphi}_F(t))} \text{ if } \varphi_F(t) \geq \hat{\varphi}_F(t) \\ 0 \text{ if } \varphi_F(t) < \hat{\varphi}^*_F(t). \end{cases} \tag{24}$$

The definition of the equilibrium distributions imply that the expression for the average productivity levels for Home- and Foreign-owned firms operating in the Home country, $\bar{\varphi}_H(t)$ and $\bar{\varphi}_F(t)$, can be rewritten as a function of the threshold levels, $\hat{\varphi}_H(t)$ and $\hat{\varphi}_F(t)$:

$$\begin{aligned} \bar{\varphi}_H(t) &= \bar{\varphi}(\hat{\varphi}_H(t)) = \left[\frac{1}{1-G(\hat{\varphi}_H)}\int_{\hat{\varphi}_H(t)}^{\infty} \varphi_H^{\mu-1}(t) g(\varphi) d\varphi\right]^{\frac{1}{\mu-1}} \\ \bar{\varphi}_F(t) &= \bar{\varphi}(\hat{\varphi}_F(t)) = \left[\frac{1}{1-G(\hat{\varphi}_F)}\int_{\hat{\varphi}_F(t)}^{\infty} \varphi_F^{\mu-1}(t) g(\varphi) d\varphi\right]^{\frac{1}{\mu-1}}. \end{aligned}$$

Both the Home- and Foreign-owned firms draw from an identical distribution, $g(\varphi)$. The distri-

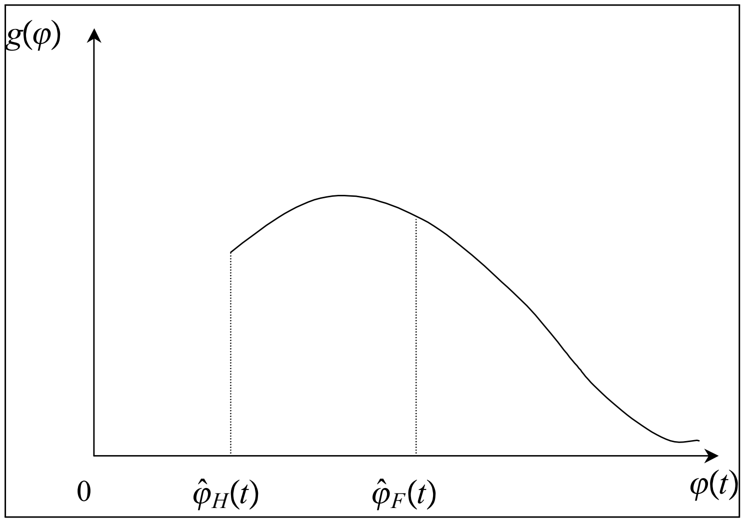

Figure 3: Probability Density of Labor Productivity in the Home Economy

bution of productivity levels in the Home economy is truncated, as depicted in Figure 3,[18] at the point $\hat{\varphi}_H(t)$ for Home-owned firms and $\hat{\varphi}_F(t)$ for Foreign-owned firms. The points of truncation will be the same or different depending on how easy it is for native firms to enter relative to foreign firms, an issue which is discussed in detail in Section 4. The aggregate productivity level in the Home country, equation (21), is thus a function of the sectoral cutoff productivity levels.

5 The Zero-Cutoff Profit Condition

Two constraints governing firm entry allow one to solve for the threshold productivity levels, $\hat{\varphi}_H(t)$ and $\hat{\varphi}_F(t)$. Threshold productivity levels are found at the point where a firm is just productive enough that its expected discounted profits equal zero. If expected discounted profits are lower, agents considering engaging in production activities will use funds they might have sunk into the fixed cost for next-period production to invest instead in bonds or to increase their present consumption. To determine the equilibrium solution, characterized by $\{P_t, \hat{\varphi}_H(t), \hat{\varphi}_F(t)\}$, one can use the zero-profit conditions (ZCPs) governing investment in the Home economy,

$$E_{t-1}[d_t \pi_H(\hat{\varphi}_H(t))] = 0$$

[18]Figure 3 is an illustration supposing that $g(\varphi)$ is some distribution resembling a normal distribution. This does not have to be the case. It is merely required that the distribution have a finite $\mu - 1$-degree moment (that the "$\mu - 1$th" moment be finite– see Melitz (2002)).

for Home-owned firms and

$$E_{t-1}[d_t^* \pi_F(\hat{\varphi}_F(t))] = 0$$

for Foreign-owned firms.

First, it is useful to express $E[d_t \pi_H(\hat{\varphi}_H(t))]$ and $E[d_t^* \pi_F(\hat{\varphi}_H^*(t))]$ as functions of revenue. Beginning with the definition of domestic profits and making the appropriate substitutions using the wage relation and consumption equation (expressions (5) and (8)), expected discounted profits from Home-country sales can be written[19]

$$\begin{aligned} E_{t-1}[d_t \pi_H(\varphi_H(t))] &= E_{t-1}[d_t(p_H(\varphi_H(t))c_H(\varphi_H(t)) - W_t l_H(\varphi_H(t)) - P_t f)] \\ &= E_{t-1}[d_t(1 - \alpha E\left[\frac{1}{M_t}\right] M_t) r_H(\varphi_H(t))] - f P_t E_{t-1}[d_t]. \end{aligned} \tag{25}$$

For prospective Foreign entrants, the expression for expected discounted profits from Home-market sales is

$$E_{t-1}[d_t^* \pi_F(\varphi_H(t))] = E_{t-1}[d_t^* \left(\frac{1}{S_t}\right)(1 - \alpha E_{t-1}\left[\frac{1}{M_t}\right] M_t) r_F(\varphi_F(t))] - \left(\frac{1}{S_{t-1}}\right) P_t f E_{t-1}[d_t^*]. \tag{26}$$

The equations for expected discounted profits are very similar for Home- and Foreign-owned firms. The principle differences arise from two points: (1) the respective discount factors, which are rooted in the monetary conditions expected to emerge in each firm owner's *native* country[20] and (2) the explicit introduction of the exchange rate, $\frac{1}{S_t}$ and $\frac{1}{S_{t-1}}$, into the Foreign firm's calculation of expected revenues. If the exchange rate were fixed and conditions in both countries were governed by a common monetary innovation or monetary authority, a Home and Foreign firm's expected discounted profits from sales in the Home-country market would be distinguishable only by their unique productivity levels, $\varphi_H(t)$ and $\varphi_F(t)$.

Setting (25) and (26) equal to zero and using the equations for prices and aggregate consumption

[19] See Appendix C.2 for detailed derivation.

[20] Explicitly, the discount factor for residents of the Home country is $E_{t-1}[d_t] = E_{t-1}[\frac{\beta P_{t-1} C_{t-1}^{\rho}}{P_t C_t^{\rho}}] = \beta E_{t-1}[\frac{M_{t-1}}{M_t}]$, whereas the discount factor for residents of the Foreign country is $E_{t-1}[d_t^*] = E_{t-1}[\frac{\beta P_{t-1}^* C_{t-1}^{*\rho}}{P_t^* C_t^{*\rho}}] = \beta E_{t-1}[\frac{M_{t-1}^*}{M_t^*}]$.

derived above, the ZCP conditions yield expressions for the threshold productivity levels,[21]

$$\hat{\varphi}_H(t) = \left(\frac{f E_{t-1}[\frac{M_{t-1}}{M_t}]}{a_1 M_{t-1} E_{t-1}\left[\left(\frac{1}{M_t}\right)^{\frac{\rho-1}{\rho}} \left(1 - \alpha E_{t-1}\left[\frac{1}{M_t}\right] M_t\right)\right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}} \tag{27}$$

$$\hat{\varphi}_F(t) = \left(\frac{f_{MNE} E_{t-1}[\frac{M^*_{t-1}}{M^*_t}]}{a_1 M_{t-1} E_{t-1}\left[\left(\frac{1}{M_t}\right)^{\frac{\rho-1}{\rho}} \left(1 - \alpha E_{t-1}\left[\frac{1}{M_t}\right] M_t\right)\right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}}, \tag{28}$$

where $a_1 = N^{\frac{1-\rho(\mu-1)}{\rho(\mu-1)}} \left(\frac{\alpha E_{t-1}[\frac{1}{M_t}]}{\kappa}\right)^{\frac{1}{\rho}}$. Dividing $\hat{\varphi}_H(t)$ by $\hat{\varphi}_F(t)$ provides a measure of Foreign investors' relative willingness to invest in the Home economy:

$$\frac{\hat{\varphi}_F(t)}{\hat{\varphi}_H(t)} = \left(\frac{f_{MNE} E_{t-1}[\frac{M^*_{t-1}}{M^*_t}]}{f E_{t-1}[\frac{M_{t-1}}{M_t}]} \right)^{\frac{1}{\mu-1}}. \tag{29}$$

Equation (29) relates the threshold productivity level of Foreign firms operating in the Home economy to that of the Home firms in terms of the fundamental variables, M and M^*; the fixed costs, f and f_{MNE}; and the elasticity of substitution, μ.

In addition to permitting equation (27) to be solved for $\hat{\varphi}_H$,[22] equation (29) provides an expression for the ratio of the productivity levels of the least productive Home and Foreign firm, which allows an investigation into the effect of changes in underlying parameters on the relative difficulty Foreign agents face when investing in the Home country. Let the ratio be designated γ. Since the money-supply growth process is assumed to be stationary, γ is a constant given by

$$\begin{aligned} \gamma &= \frac{\hat{\varphi}_F(t)}{\hat{\varphi}_H(t)} = \left(\frac{f_{MNE}}{f}\right)^{\frac{1}{\mu-1}} \left(\frac{e^{\sigma^2_{m^*}}}{e^{\sigma^2_m}}\right)^{\frac{1}{\mu-1}} \\ &= \left[\left(\frac{(1+\psi)}{(1+\psi^*)}\right)\left(\frac{f_{MNE}}{f}\right)\right]^{\left(\frac{1}{\mu-1}\right)} e^{\left(\frac{1}{\mu-1}\right)\left(\sigma^2_{m^*} - \sigma^2_m\right)}. \end{aligned} \tag{30}$$

If $\gamma = 1$, then Home and Foreign firms have equal access to the Home market. As γ increases, more Home firms relative to Foreign firms expect entry to be profitable, meaning that it is harder for Foreign firms than native firms to enter the Home market. A casual look at (30) indicates

[21] See Appendix C.3 for detailed derivation.
[22] See Appendix C.4 for proof.

that raising the fixed cost incurred by Foreign entrants relative to that paid by domestically owned entrants increases γ, reflecting the increased difficulty Foreign firms would face relative to Home firms when entering the Home market. The effects of the mean and variance of the money-supply growth rates, as well as the intuition behind them, are discussed in Part 4.

5.1 A Note on Complete Markets, Factor-Price Equalization, and Geographic Preference

The availability of a complete set of state-contingent bonds allows consumers to insure against country-specific risk, so that marginal utility is equal across countries in any given period. As Devereux and Engel (2001) point out, this risk sharing results in factor-price equalization– wages will be equal in the Home and Foreign countries. To see that this is true here also, one can examine the wage relation for each country's representative consumer (equation (5) and its Foreign-country counterpart, $W_t^* = P_t^* C_t^{*\rho}$), along with the formula for the real exchange rate (10):

$$W_t = \kappa P_t C_t^\rho = \frac{\kappa P_t C_t^\rho}{\kappa P_t^* C_t^{*\rho}} \kappa P_t^* C_t^{*\rho} = S_t W_t^*.$$

Because the wage is equal across countries, the attractiveness of investing in one's native market as compared with overseas is determined solely by the relative costs of entry. As long as $\gamma \geq 1$ (i.e., $\hat{\varphi}_F(t)$ is at least as large as $\hat{\varphi}_H(t)$), which is assumed in the following analysis, there will be Home firms which do not invest in the Foreign market and Foreign firms which do not invest in the Home market. Foreign firms will always choose to invest in the Foreign country, with some fraction of the entrants also choosing to invest in the Home country. It is noted here that if the stochastic processes governing the growth rate of the money supply in each country are identical (that is, $\psi = \psi^*$ and $\sigma_m^2 = \sigma_{m^*}^2$), then $\gamma \geq 1$ as long as $f_{MNE} \geq f$. Intuitively speaking, firms will always invest in their native country before investing abroad as long as the fixed entry cost overseas does not look too "cheap" relative to the cost of entering their own native market. This assumption adheres to the spirit of Dunning's argument that there is an additional "cost of doing business abroad" associated with overseas operations (Dunning 1973). Relaxing this assumption in a setting with incomplete markets and/or real wage rigidity would lead to a model of geographic preference, which is ground for further research.

6 The Effect of Exchange-Rate Uncertainty on Entry by Foreign Firms

As mentioned above, the relationship between the threshold productivity levels of Home and Foreign firms operating in the Home market reveals the effect of the fundamental variables and the structural parameters governing demand on the relative willingness of Foreign investors to engage in ventures overseas. Another way to look at γ is as a parameter embodying the effect of exchange-rate risk on entry by Foreign firms. Rearranging (30),

$$\begin{aligned}\hat{\varphi}_F(t) &= \gamma\hat{\varphi}_H(t) \\ &= \left[\left(\frac{(1+\psi)}{(1+\psi^*)}\right)\left(\frac{f_{MNE}}{f}\right)\right]^{\left(\frac{1}{\mu-1}\right)} e^{\left(\frac{1}{\mu-1}\right)\left(\sigma^2_{m^*}-\sigma^2_m\right)}\hat{\varphi}_H(t),\end{aligned}$$

it is evident that the minimum productivity level for Foreign entrants into the Home economy is composed of two effects. The first effect, embodied in $\hat{\varphi}_H(t)$, reflects the influence of any factor that would create a more or less welcoming environment for any investor contemplating a startup in the Home country–affecting all firms, both Home- and Foreign-owned, in the same manner. The second, contained in γ, represents the influence of variables that impact Foreign-owned firms differently than domestically owned firms due to the exchange-rate risk incurred when investors pay the fixed overhead cost in period $t-1$ required to start production in period t. The term γ reflects the size of the sunk cost for Foreign firms relative to that of domestic firms, $\frac{f_{MNE}}{f}$. However, if this consideration is neutralized by letting $f_{MNE} = f$, then γ represents only the net effect of expected changes in monetary variables across the two countries, $\left(\frac{(1+\psi)}{(1+\psi^*)}\right)^{\left(\frac{1}{\mu-1}\right)} e^{\left(\frac{1}{\mu-1}\right)\left(\sigma^2_{m^*}-\sigma^2_m\right)}$, which Foreign producers in the Home country care about because they pay the fixed cost valued at rate $\frac{1}{S_{t-1}}$, but repatriate profits at the rate $\frac{1}{S_t}$.

6.1 Exchange Rate Risk and the Host Country's Money-Supply Growth Rate

The two separate effects at work in determining the cutoff level of productivity for Foreign firms in the Home market are quite distinct and can actually exert opposing influences on Foreign entry. An increase in Home money-supply volatility decreases the value of γ,

$$\frac{\partial\gamma}{\partial\sigma^2_m} = -\frac{\gamma}{(\mu-1)} < 0, \tag{31}$$

which means that the relative difficulty facing Foreign-owned firms face when entering the Home market declines, pushing down $\hat{\varphi}_F(t)$. Nonetheless, a simulation of the total effect of increasing σ^2_m

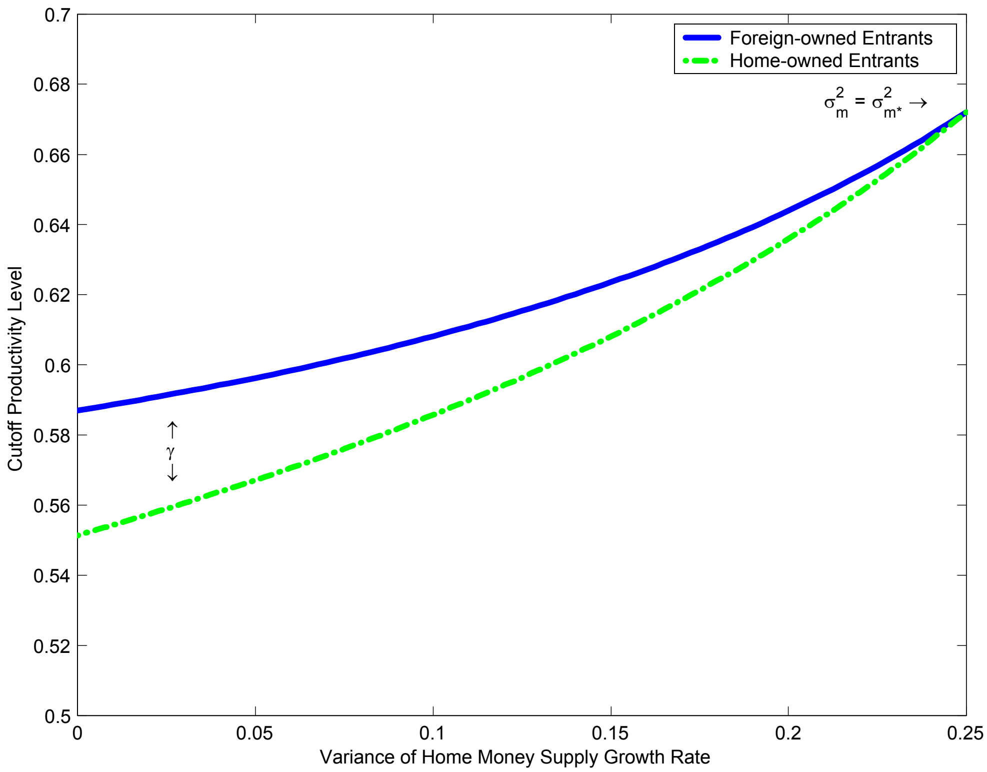

Figure 4: Volatility in the Growth Rate of the Home Money Supply and Productivity Thresholds

reveals that this does not necessarily mean that entry by Foreign firms will rise in an absolute sense. Figure 4 shows that an increase in the variance of the growth rate of the Home money supply from zero to 0.25 percent generates overall less favorable conditions for prospective investors in the Home country, illustrated by the continuous rise in $\hat{\varphi}_H(t)$ and the fall in the proportion of all prospective Home investors that choose enter the domestic market (Figure 5).[23] Yet the decreasing value of γ implies that Foreign firms operating in the Home market are protected somewhat from the perils of Home monetary uncertainty by virtue of their exchange-rate exposure. How could this happen? Even though an increase in Home monetary volatility makes investment in the Home country less inviting for all firms due to sunk costs and sticky prices, the effect is offset somewhat for Foreign firms because the threat of an unexpected fall in the growth rate of the money supply, which would

[23] See Appendices C.4 and C.5 for a discussion of restrictions on parameter values and the calibration of the model. The simulation assumes for simplicity that $f = f_{ME}$ and fixes the value of $\sigma^2_{m^*}$ at 1 to satisfy the geographic preference condition described in Section 3.3. It is interesting to note that if the real value of the fixed operating costs are the same for Home and Foreign firms in the Home country, then $\hat{\varphi}_F(t)$ is equal to $\hat{\varphi}_H(t)$ when $\sigma^2_m = \sigma^2_{m^*}$, indicated by the intersection of the two lines at $\sigma^2_m = \sigma^2_{m^*} = 1$ in the graph.

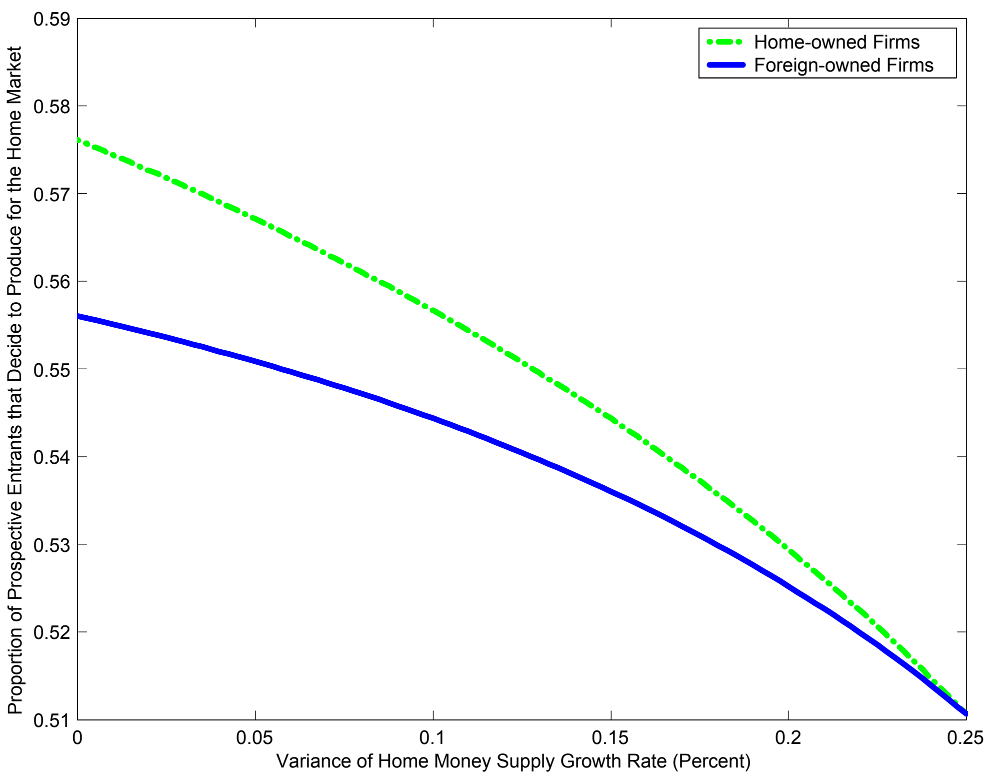

Figure 5: Volatility in the Growth Rate of the Home Money Supply and Entry

depress sales in the Home market, is cushioned by the promise of a simultaneous appreciation of the Home country's currency.

6.2 Exchange Rate Risk and the Native Country's Money-Supply Growth Rate

The two effects also work in opposite directions for changes in the volatility of Foreign money-supply growth. For increases in $\sigma^2_{m^*}$, the change in γ is positive,

$$\frac{\partial \gamma}{\partial \sigma^2_{m^*}} = \frac{\gamma}{(\mu - 1)} > 0. \tag{32}$$

Exchange rate fluctuations generated by shocks to the Foreign money-supply growth rate are not cushioned by offsetting fluctuations in Home-country sales. Indeed, an unexpected drop in the Foreign money supply not only results in an unexpected depreciation of the Home currency, it does so at a time of unexpectedly low real income in the Foreign country, a condition that may be spread to the Home-country market, as well, through risk sharing. Thus, exchange-rate risk introduced through volatility in the growth rate of the Foreign money supply is not offset and can potentially be exacerbated by fluctuations in sales by branches of Foreign firms operating in the Home economy.

The cutoff productivity level of Foreign firms entering the Home market rises with increases in $\sigma^2_{m^*}$, both in a relative and an absolute sense. (See Figure 6.) Fewer firms of larger average size will enter, complementing Campa's (1993) finding that exchange-rate volatility can deter entry by Foreign firms. Figure 7 illustrates that adverse exchange-rate risk arising from increases in the volatility Foreign money-supply growth rate pushes the least productive Foreign-owned firms out of the Home market, even as their defection allows a very small number of less productive domestic investors to soak up their abandoned market share.[24] Also important is that this deterrent effect increases with the size of local fixed costs,

$$\frac{\partial}{\partial f_{MNE}} \left[\frac{\partial \gamma}{\partial \sigma^2_{m^*}} \right] = \frac{\gamma}{(\mu - 1)^2 f_{MNE}} > 0,$$

which coincides with the findings of Sekkat and Galgau (2001), who show that volatility is more likely to have a deterrent effect for heavy industry, which involves higher local fixed costs.

Thus, the results in (31) and (32) imply that exposure to unpredictable fluctuations in the exchange rate generated by volatility in underlying monetary variables can either encourage or discourage FDI, depending on whether the volatility comes from the host-country money supply or from a firm's native economy. This dual result offers a theoretical explanation for conflicting results

[24] In the second simulation, illustrating the behavior of the threshold productivity levels for increasing values of $\sigma^2_{M^*}$, σ^2_M is fixed at 0 to satisfy the geographic preference condition. Again, it is true that $\hat{\varphi}_F(t) = \hat{\varphi}_H(t)$ whenever $\sigma^2_m = \sigma^2_{m^*}$ (at $\sigma^2_m = \sigma^2_{m^*} = 0$, in this case).

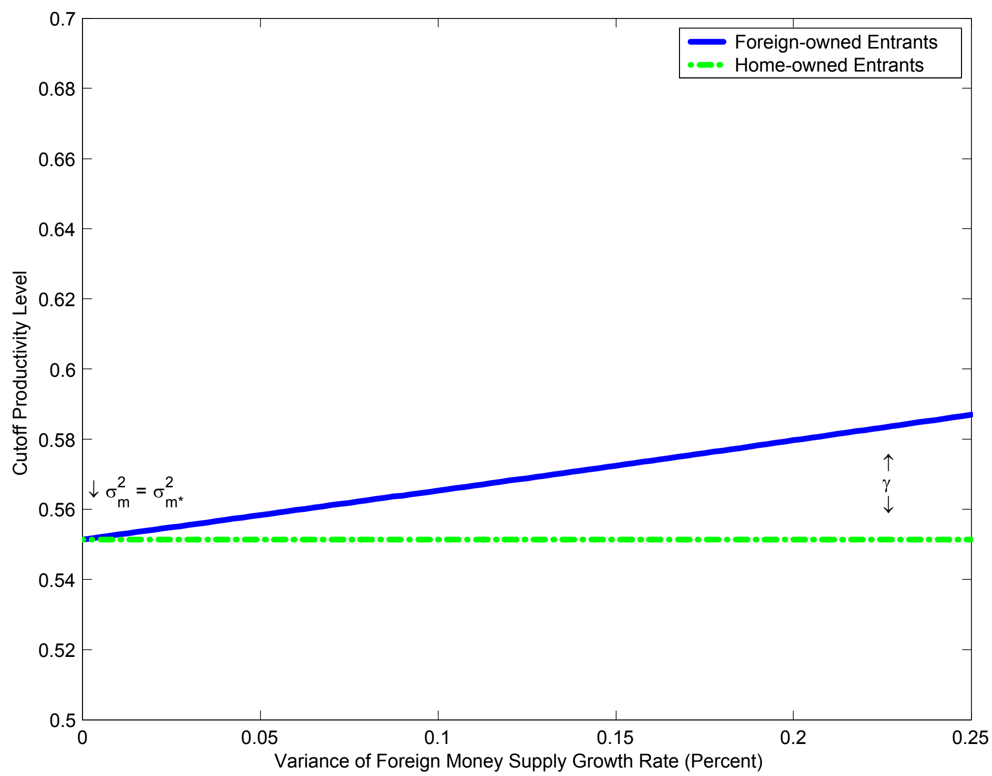

Figure 6: Volatility in the Growth Rate of the Foreign Money Supply and Productivity Thresholds

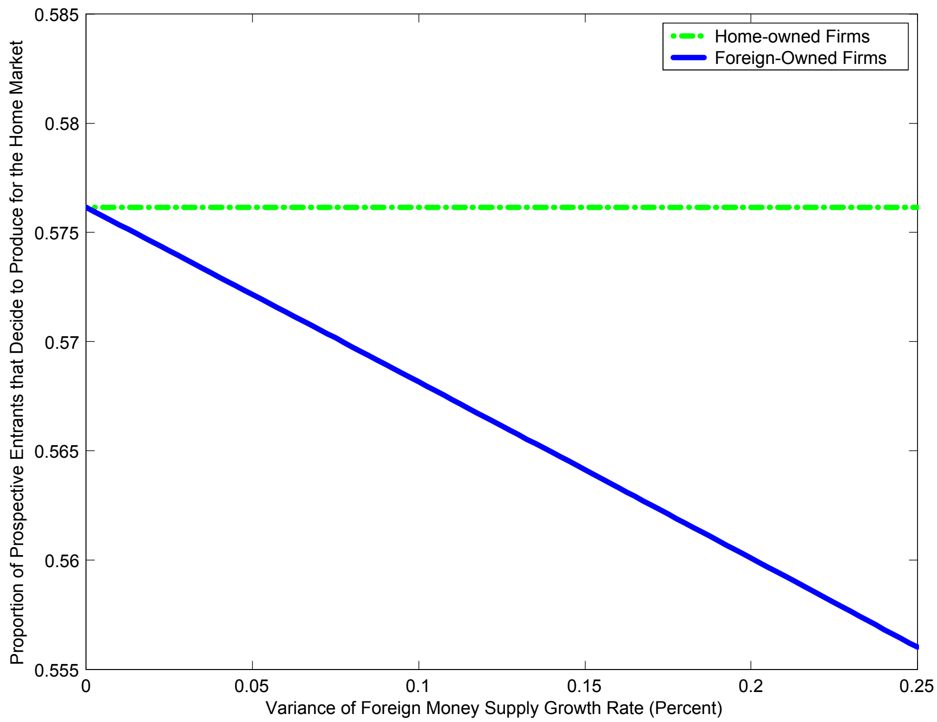

Figure 7: Volatility in the Growth Rate of the Foreign Money Supply and Entry

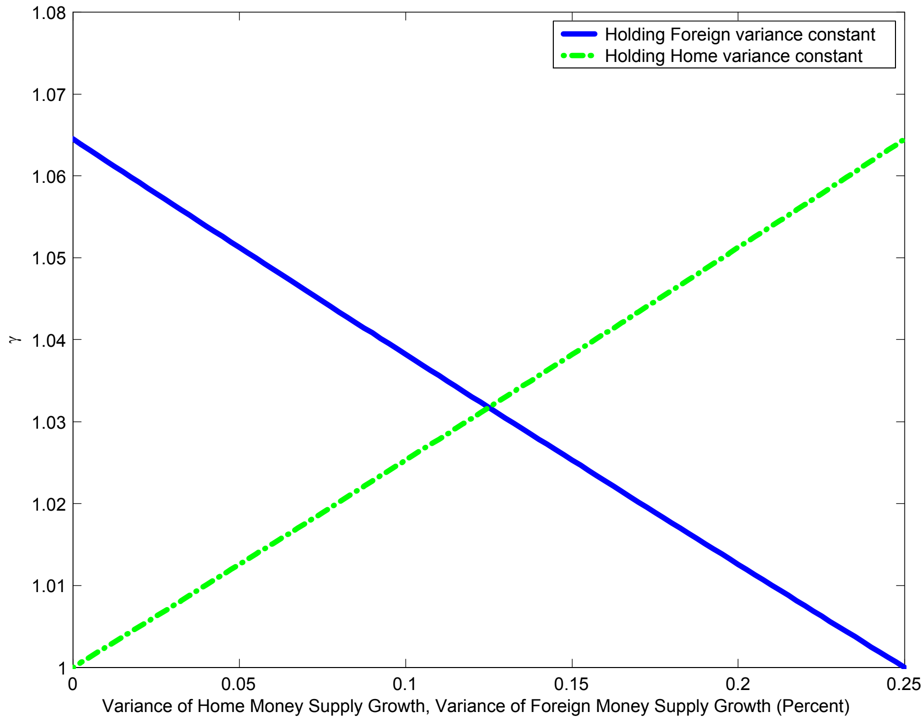

Figure 8: Opposite Effects of Home and Foreign Monetary Volatility on the Difficulty of Entry Faced by Foreign Firms

in empirical tests of partial equilibrium models. Figure 8 provides a graphical representation of this duality. The relative willingness of Foreign firms to invest in the Home economy grows as σ_m^2 increases. However, increasing $\sigma_{m^*}^2$ causes γ to rise, indicating that entry in to the Home market is less attractive to Foreign firms. Interestingly, when monetary volatility is perfectly symmetric across the two countries ($\sigma_m^2 = \sigma_{m^*}^2$), the ratio of Home and Foreign firms is not affected at all by monetary volatility– and therefore not by exchange rate uncertainty, either, in this stylized model. Further, whereas the volatility of the Home and Foreign money-supply growth rates have opposing effects on γ, they affect exchange rate volatility with the same sign,[25] bearing the implication that *there is no clear correlation between exchange rate volatility and FDI* unless one takes into account the origin of the volatility.

The factor-price equalization noted in Section 3.3 raises the question of whether the above results hold in a world with incomplete markets, where wages are not necessarily equal across countries.

[25] In this model, $var(\log S_t) = var(m_t) + var(m_t^*)$ since M and M^* are independently distributed.

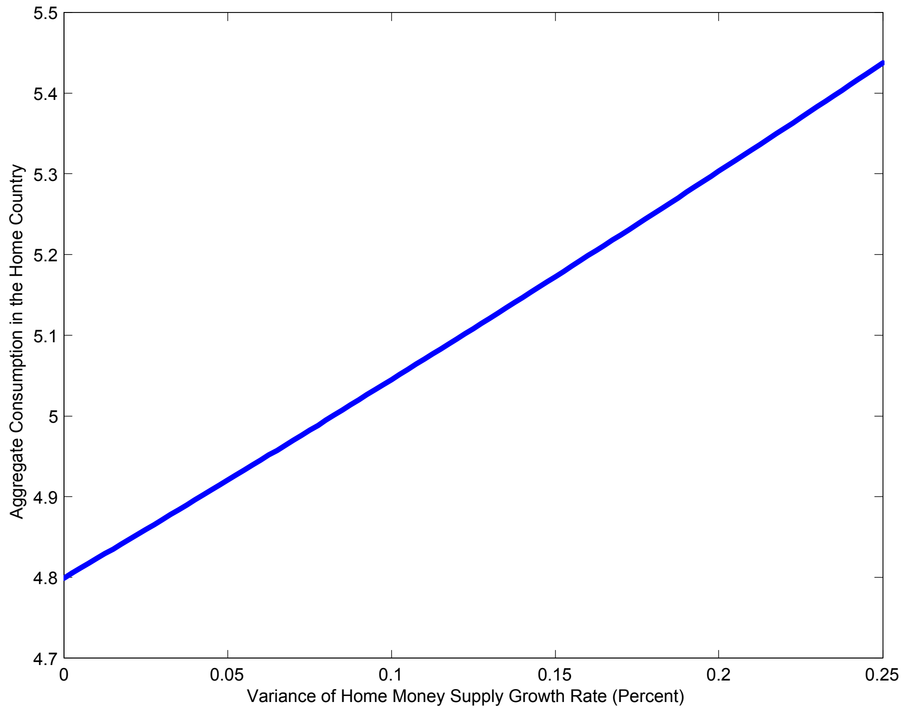

Figure 9: Aggregate Consumption and Home Monetary Volatility

Appendix D presents a model without bonds and simplified to a one-period framework where the impacts of Home and Foreign money-supply volatility are very similar to the complete markets case. Under incomplete markets, multinational corporations factor into their decisionmaking the fact that the local wage rate will change due to shocks to the money supply. However, the net effect of the uncertainty is the same as when perfect risk-sharing is possible. The results therefore appear robust to the structure of the asset market.

6.3 Prices and Consumption in the Host Country

The sectoral responses to changes in monetary volatility have an impact on aggregate productivity, the price level, and consumption in the Home country. As Home-owned firms exit more quickly than more productive Foreign-owned firms in response to increases in σ_m^2, aggregate productivity in the economy increases, generating downward pressure on the aggregate price level. The resulting increase in Home consumption is shown in Figure 9. Conversely, when increases in Foreign monetary volatility drive Foreign-owned firms out of the Home market, to be partially replaced by less productive Home-owned firms entering to capture a bit of the abandoned market share, there

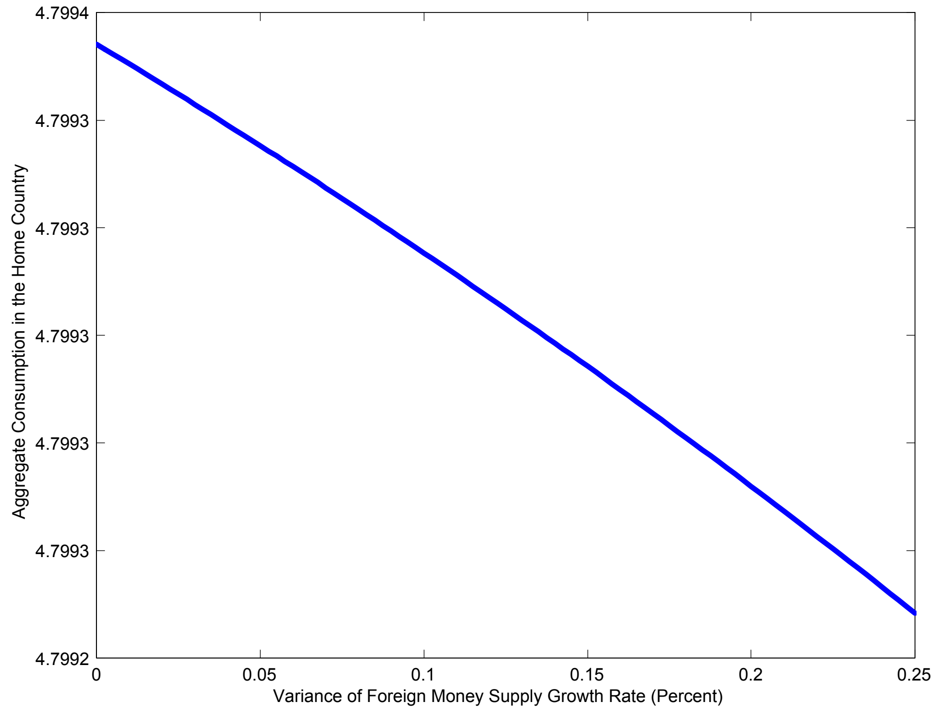

Figure 10: Aggregate Consumption and Foreign Monetary Volatility

is a mild downward impact on the aggregate productivity level, pushing up prices and exerting a minute negative effect on Home consumption, shown in Figure 10.

7 Conclusions

The goal of this paper is to explain the conflicting findings of previous empirical work done in a partial equilibrium framework by showing that volatility in the exchange rate may or may not deter foreign direct investment, depending on which underlying variable is the source of the volatility. The result here provides a theoretical account of the link between FDI flows and the correlation between local demand and exchange-rate volatility investigated by Goldberg and Kolstad (1995). It bears the important and empirically testable implication that the variance of the exchange rate will impact the MNE's decision to enter a market, but whether it encourages or deters firms contemplating direct investment depends on whether the shocks originate in the company's own native country or overseas, in the host market. As described by Campa (1993), the extent to which MNEs worry about exchange rate volatility is closely related to the presence and magnitude of local sunk costs. Thus, in several ways, the model and its results are an extension of prior investigations of the determinants of direct investment in the trade and industrial-organization literature.

The findings presented here also contribute to the literature of open-economy macroeconomics on three fronts. First, they echo a key point in Melitz (2003) that a country's aggregate level of labor productivity can change without any change in available technology, characterized here by $g(\varphi)$. Whereas Melitz shows that these changes can occur in response to changes in fixed costs and trade patterns, this paper introduces the effect of the first and second moments of the growth rate of the money supply, variables controlled by monetary policymakers. Second, if one envisions production by multinationals for local consumption as a case of pricing-to-market, as Devereux and Engel (2001) propose, then the responses of both the aggregate price level and consumption to volatility in the growth rate of the Foreign money supply demonstrated above– even though they are extremely small– as well as the redistribution of production from the Foreign-owned to the Home-owned sector, are a corollary to the conventional wisdom that pricing-to-market insulates an economy from shocks to the money supply in other countries of the world.[26] Finally, the model suggests that both the behavior of overseas investors and the level and volatility of exchange rates may be jointly determined by common underlying macroeconomic variables, implying that regressions of FDI flows on both movements in exchange rate levels and on proxies for exchange rate uncertainty, such as its variance, are subject to the same types of endogeneity issues as studies of the impact of exchange rate uncertainty on trade flows.[27]

The results point to several avenues for future research. It would be informative to introduce trade or vertical FDI into a version of the model with incomplete markets to look at the effect of exchange rate uncertainty and fluctuations in local costs of production on the concentration of productive capacity, in the spirit of nonmonetary models by Brainard (1997); Helpman, Melitz, and Yeaple (2003); and Markusen and Venables (2002). It would also be useful to examine the effect of productivity shocks as a source of macroeconomic uncertainty. In addition, the incorporation of physical capital into the model might illuminate the impact of exchange-rate variablity on the growth of a country's capital stock over time.

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[27] Wang and Barrett (2002) and Tenreyro (2003) are two examples.

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Appendix A: Deriving the Intratemporal Demand Equations

The representative consumer in the Home country maximizes lifetime utility subject to an intertemporal budget constraint:

$$\max E_t\left[\sum_{t=0}^{\infty}\beta^t U_t\right]$$

$$s.t.\ P_tC_t+M_t+\sum_{z^{t+1}|z^t} q(z^{t+1}|z^t)B(z^{t+1})=W_tL_t+\pi_t+M_{t-1}+B_t+T_t$$

where U is a function of aggregate consumption, C, and labor, L,

$$U_t=\frac{1}{1-\rho}C_t^{1-\rho}+\chi\ln\left(\frac{M_t}{P_t}\right)-\kappa L_t,$$

and $q(z^t|z^{t-1})$ is the price at time $t-1$ of the bond $B(z^t)$, which is denominated in Home currency and has a payoff of one unit of home currency given that one of a set z of possible states of the macroeconomy is realized at the end of time t.[1] Aggregate consumption is an index reflecting preferences with constant elasticity of substitution (CES) across the set of all available goods,

$$C_t=\left[\int_0^{n_H} c_H(i,t)^{\frac{\mu-1}{\mu}}di+\int_1^{1+n_F} c_F(i,t)^{\frac{\mu-1}{\mu}}di\right]^{\frac{\mu}{\mu-1}}.$$

A.1. The Aggregate Price Index

The aggregate price index is defined as the minimum expenditure required to purchase one unit of the consumption index, C_t. To find P_t, one can solve the following minimization problem:[2]

$$\begin{aligned}\pounds &= \min\int_0^{n_H} p_H(i,t)c_H(i,t)di+\int_1^{1+n_F} p_F(i,t)c_F(i,t)di\\ &\quad+\psi_t\left(1-\left[\int_0^{n_H} c_H(i,t)^{\frac{\mu-1}{\mu}}di+\int_1^{1+n_F} c_F(i,t)^{\frac{\mu-1}{\mu}}di\right]^{\frac{\mu}{\mu-1}}\right),\end{aligned}$$

where ψ_t is the Lagrange multiplier. First-order conditions:

$$\frac{\partial\pounds}{\partial c_H(i,t)}: p_H(i,t)-\psi_t\left[\int_0^{n_H} c_H(i,t)^{\frac{\mu-1}{\mu}}di+\int_1^{1+n_F} c_F(i,t)^{\frac{\mu-1}{\mu}}di\right]^{\frac{1}{\mu-1}} c_H(i,t)^{\frac{-1}{\mu}}=0 \qquad \text{(a.1)}$$

$$\frac{\partial\pounds}{\partial c_F(i,t)}: p_F(i,t)-\psi_t\left[\int_0^{n_H} c_H(i,t)^{\frac{\mu-1}{\mu}}di+\int_1^{1+n_F} c_F(i,t)^{\frac{\mu-1}{\mu}}di\right]^{\frac{1}{\mu-1}} c_F(i,t)^{\frac{-1}{\mu}}=0 \qquad \text{(a.2)}$$

[1] That is, $\sum_{z^{t+1}|z^t} B(z^{t+1})$ represents a complete set of state-contingent bonds denominated in the Home currency.

[2] See Obstfeld and Rogoff (1996, Chapter 4).

$$\frac{\partial \pounds}{\partial \psi_t} : 1 - \left[\int_0^{n_H} c_H(i,t)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F} c_F(i,t)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}} = 0 \tag{a.3}$$

Rearranging (a.1),

$$\begin{aligned} p_H(i,t) - \psi_t C_t^{\frac{1}{\mu}} c_H(i,t)^{\frac{-1}{\mu}} &= 0 \\ \frac{1}{\psi_t} p_H(i,t) C_t^{\frac{-1}{\mu}} &= c_H(i,t)^{\frac{-1}{\mu}} \\ c_H(i,t) &= \left(\frac{1}{\psi_t}\right)^{-\mu} p_H(i,t)^{-\mu} C_t. \end{aligned}$$

By definition, to find P_t, $C_t = 1$, so that

$$c_H(i,t) = \psi_t^{\mu} p_H(i,t)^{-\mu}. \tag{a.4}$$

Similarly, a counterpart equation for $c_F(i,t)$ emerges,

$$c_F(i,t) = \psi_t^{\mu} p_F(i,t)^{-\mu}. \tag{a.5}$$

Substituting (a.4) and (a.5) into (a.3) it is possible to solve for ψ_t^{μ}:

$$\begin{aligned} 1 - \left(\int_0^{n_H} \left(\psi_t^{\mu} p_H(i,t)^{-\mu}\right)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F} \left(\psi_t^{\mu} p_F(i,t)^{-\mu}\right)^{\frac{\mu-1}{\mu}} di \right)^{\frac{\mu}{\mu-1}} &= 0 \\ \psi_t^{\mu} \left(\int_0^{n_H} \left(p_H(i,t)^{-\mu}\right)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F} \left(p_F(i,t)^{-\mu}\right)^{\frac{\mu-1}{\mu}} di \right)^{\frac{\mu}{\mu-1}} &= 1 \\ \left(\begin{array}{c} \int_0^{n_H} \left(p_H(i,t)^{-\mu}\right)^{1-\mu} di \\ + \int_1^{1+n_F} \left(p_F(i,t)^{-\mu}\right)^{1-\mu} di \end{array} \right)^{\frac{1}{1-\mu}} &= \psi_t \end{aligned}$$

Since ψ_t, the Lagrangian multiplier, represents the value of one extra unit of the consumption index, C_t, it is by definition an expression of the aggregate price index. Therefore, the two are equal:

$$P_t = \psi_t = \left(\int_0^{n_H} \left(p_H(i,t)\right)^{1-\mu} di + \int_1^{1+n_F} \left(p_F(i,t)\right)^{1-\mu} di \right)^{\frac{1}{1-\mu}}. \tag{a.6}$$

A.2 The Demand for an Individual Good

To determine the demand for $c_H(i,t)$ when $C_t = 1$, one can isolate ψ_t from () to obtain the relation

$$P_t = \psi_t = p_H(i,t) c_H(i,t)^{\frac{1}{\mu}},$$

yielding the demand equation

$$c_H(i,t) = \left(\frac{p_H(i,t)}{P_t} \right)^{-\mu}.$$

Since CES preferences are homothetic, the demand for any particular good is a constant proportion of aggregate consumption. Therefore, if it is true that

$$\frac{c_H(i,t)}{C_t} = \left(\frac{p_H(i,t)}{P_t} \right)^{-\mu}$$

for the case where $C_t = 1$, the ratio must be constant for all levels of aggregate consumption. Thus, the demand for an individual good is given by

$$c_H(i,t) = \left(\frac{p_H(i,t)}{P_t} \right)^{-\mu} C_t. \tag{a.7}$$

An analogous equation holds for individual foreign goods produced in the home country:

$$c_F(i,t) = \left(\frac{p_F(i,t)}{P_t} \right)^{-\mu} C_t$$

The demand relation also implies that expenditure on an individual home or foreign good will be proportional to total consumer expenditure. Multiplying by $p_H(i,t)$,

$$\begin{aligned} p_H(i,t) c_H(i,t) &= p_H(i,t)^{1-\mu} P_t^{\mu-1} P_t C_t \\ &= \left(\frac{p_H(i,t)}{P_t} \right)^{1-\mu} P_t C_t. \end{aligned}$$

Appendix B: First-Order Conditions and the Exchange Rate

Given the consumer's maximization problem above, let λ be the Lagrange multiplier for the intertemporal budget constraint.

$$\frac{\partial \pounds}{\partial C_t} : C_t^{-\rho} - \lambda_t P_t = 0 \Rightarrow \lambda_t = \frac{1}{P_t C_t^{\rho}} \tag{b.1}$$

$$\frac{\partial \pounds}{\partial M_t} : \frac{\chi}{M_t} - \lambda_t + \beta E_t \left[\lambda_{t+1} \right] = 0 \tag{b.2}$$

$$\frac{\partial L}{\partial L_t} : -\kappa + \lambda_t W_t = 0 \tag{b.3}$$

$$\frac{\partial \pounds}{\partial B_{t+1}} : \beta E_t \left[\lambda_{t+1} \right] - q(z^{t+1} | z^t) \lambda_t = 0 \tag{b.4}$$

Substituting (b.1) into (b.3) provides the wage relation (also equation (4))

$$W_t = \kappa P_t C_t^{\rho}. \tag{b.5}$$

Substituting (b.1) into (b.2) and rearranging yields the demand equation (equation (5) in the main text) for real money balances:

$$\begin{aligned} \frac{\chi}{M_t} &= \frac{1}{P_t C_t^{\rho}} - \beta E_t \left[\frac{1}{P_{t+1} C_{t+1}^{\rho}} \right] \\ \frac{\chi P_t C_t^{\rho}}{M_t} &= 1 - E_t \left[\frac{\beta P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}} \right] \\ \frac{M_t}{P_t} &= \frac{\chi C_t^{\rho}}{1 - d_{t+1}}, \end{aligned} \tag{b.6}$$

where $d_{t+1} = E_t \left[\frac{\beta P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}} \right]$. Again substituting (b.1) into (b.4) gives a bond-pricing equation,

$$q(z^{t+1}|z^t) = E_t \left[\frac{\beta P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}} \right], \tag{b.7}$$

B.1 The Foreign Consumer

The Foreign consumer's intertemporal budget constraint will be similar to that of the Home consumer, namely

$$P_t^* C_t^* + M_t^* + \frac{1}{S_t} \sum_{z^{t+1}|z^t} q(z^{t+1}|z^t) B^*(z^{t+1}) = W_t^* L_t^* + \pi_t^* + M_{t-1}^* + S_t B_t^* + T_t^*.$$

As mentioned in the text, S_t is the nominal exchange rate at time t. Thus, the Foreign consumer's problem yields first-order conditions

$$W_t^* = \kappa P_t^* C_t^{*\rho}, \tag{b.8}$$

$$\frac{M_t^*}{P_t^*} = \frac{\chi C_t^{*\rho}}{1 - d_{t+1}^*}, \tag{b.9}$$

$$q(z^{t+1}|z^t) = \beta \left(\frac{S_t}{S_{t+1}} \right) \left(\frac{P_t^* C_t^{*\rho}}{P_{t+1}^* C_{t+1}^{*\rho}} \right). \tag{b.10}$$

B.2 Solving for the Exchange Rate

Arbitrage forces the price of a bond, $q(z^{t+1}|z^t)$ to be equal in both countries. Setting (b.7) equal to (b.10), both iterated backward, we have

$$\frac{P_{t-1} C_{t-1}^{\rho}}{P_t C_t^{\rho}} \equiv \left(\frac{S_{t-1}}{S_t} \right) \left(\frac{P_{t-1}^* C_{t-1}^{*\rho}}{P_t^* C_t^{*\rho}} \right).$$

One can iterate each side backward one period and multiply,

$$\begin{aligned}\left(\frac{P_{t-1}C_{t-1}^{\rho}}{P_tC_t^{\rho}}\right)\left(\frac{P_{t-2}C_{t-2}^{\rho}}{P_{t-1}C_{t-1}^{\rho}}\right) &= \left(\frac{S_{t-1}}{S_t}\right)\left(\frac{P_{t-1}^*C_{t-1}^{*\rho}}{P_t^*C_t^{*\rho}}\right)\left(\frac{S_{t-2}}{S_{t-1}}\right)\left(\frac{P_{t-2}^*C_{t-2}^{*\rho}}{P_{t-1}^*C_{t-1}^{*\rho}}\right) \\ \frac{P_{t-1}C_{t-1}^{\rho}}{P_tC_t^{\rho}} &= \left(\frac{S_{t-2}}{S_t}\right)\left(\frac{P_{t-2}^*C_{t-2}^{*\rho}}{P_t^*C_t^{*\rho}}\right).\end{aligned}$$

Repeating this step continually yields the expression[3]

$$\frac{P_0C_0^{\rho}}{P_tC_t^{\rho}} = \left(\frac{S_0}{S_t}\right)\left(\frac{P_0^*C_0^{*\rho}}{P_t^*C_t^{*\rho}}\right),$$

or

$$S_t = S_0\left(\frac{P_0^*C_0^{*\rho}}{P_t^*C_t^{*\rho}}\right)\left(\frac{P_tC_t^{\rho}}{P_0C_0^{\rho}}\right).$$

Assuming that both the Home and Foreign country have identical initial conditions, so that $S_0P_0^*C_0^{*\rho} = P_0C_0^{\rho}$, one obtains an equation for the real exchange rate, equation (19) in the text,

$$\frac{S_tP_t^*}{P_t} = \frac{C_t^{\rho}}{C_t^{*\rho}}. \tag{b.11}$$

Rearranging (b.6) and (b.9), one obtains expressions for consumption in each country,

$$C_t^{\rho} = \frac{M_t}{P_t}\left(\frac{1-d_{t+1}}{\chi}\right) \qquad C_t^{*\rho} = \frac{M_t^*}{P_t^*}\left(\frac{1-d_{t+1}^*}{\chi}\right). \tag{b.12}$$

Using the specification for the money supply growth rate described in Section 2.2,

$$E_t\left[\frac{M_t}{M_{t+1}}\right] = \theta \qquad E_t\left[\frac{M_t^*}{M_{t+1}^*}\right] = \theta^*,$$

(so that $\theta = \frac{e^{\sigma_m^2}}{(1+\psi)}$) Obstfeld and Rogoff (1998, p.39) show that consumption is a function of real money balances and the underlying parameters,

$$C_t^{\rho} = \frac{M_t}{P_t}\left(\frac{1-\beta\theta}{\chi}\right) \qquad C_t^{*\rho} = \frac{M_t^*}{P_t^*}\left(\frac{1-\beta\theta^*}{\chi}\right). \tag{b.13}$$

Subsituting (b.13) into (b.11), an expression for the nominal exchange rate emerges,

$$S_t = \frac{M_t(1-\beta\theta)}{M_t^*(1-\beta\theta^*)}$$

Appendix C: Aggregation and Equilibrium Conditions

C.1.1 Revenues, Profits, and Aggregate Employment

[3] This solution process is described by Chari, Kehoe, and McGrattan (2002).

Like the price level, total revenues and total profits for firms operating on Home-country soil can be written as a function of aggregate sectoral productivity. The actual revenue, on average, for a firm picked at random from firms owned by country j producing inside the Home economy at the end of period t can be computed as

$$\bar{r}_j(t) = \int_0^\infty r_j(\varphi_j(t))\eta_j\left(\varphi(t)\right)d\varphi$$

Using expression (18) and the definition of $\bar{\varphi}_{jt}$ in (21), this simplifies as shown in Melitz (2003):

$$\begin{aligned}
\bar{r}_j(t) &= \int_0^\infty r_j(\bar{\varphi}_j(t))\frac{r_j(\varphi_j(t))}{r_j(\bar{\varphi}_j(t))}\eta_j\left(\varphi(t)\right)d\varphi. \\
&= r_j(\bar{\varphi}_j(t))\int_0^\infty \left(\frac{\varphi_j(t)}{\bar{\varphi}_j(t)}\right)^{\mu-1}\eta_j\left(\varphi(t)\right)d\varphi \\
&= r_j(\bar{\varphi}_j(t))\left(\frac{1}{\bar{\varphi}_j(t)}\right)^{\mu-1}\int_0^\infty \varphi_j(t)^{\mu-1}\eta_j\left(\varphi(t)\right)d\varphi \\
&= r_j(\bar{\varphi}_j(t)).
\end{aligned}$$

To compute total revenues as a function of aggregate sectoral productivity, it is possible to treat all firms in the Home economy owned by residents of country j as though they earned the average level of revenue in the sector of all j-owned firms,

$$\begin{aligned}
R_t &= \int_0^{n_H} r_H(\bar{\varphi}_H(t))di + \int_1^{1+n_F} r_F(\bar{\varphi}_F(t))di \\
&= n_H r_H(\bar{\varphi}_H(t)) + n_F r_F(\bar{\varphi}_F(t)).
\end{aligned}$$

Labor enters linearly in the representative consumer's utility function, making the supply of labor perfectly inelastic with respect to the wage, so that equilibrium in the labor market is fully characterized by the cutoff productivity levels and the (exogenous) parameters underlying the model. If countries are not identical, the labor market equilibrium can be calculated numerically once the cutoff productivity levels are determined. For the special case in which countries are identical, an equation depicting employment as a function of aggregate consumption can be derived analytically as follows:

Let the total level of profit earned by all firms (Home- and Foreign-owned) operating in the Home economy be denoted by Π. Appendix C.1.2 shows that using a similar process, Π_t is given by

$$\Pi_t = n_H\pi_H(\bar{\varphi}_H(t)) + n_F\pi_F(\bar{\varphi}_F(t)).$$

Suppose that the labor force is composed of manufacturing workers and entrepreneurial workers, who are engaged in investment and managerial activities. Then aggregate expenditure on manufacturing labor in the Home economy equals total revenues, less the profits distributed to entrepreneurial workers,

$$W_t L_t^P = R_t - \Pi_t,$$

where L_t^P is the amount of labor hired in the production of goods in the Home economy. Profits are distributed to entrepreneurial workers at the same wage rate,[4] so that if L_t^E is the level of entrepreneurial labor hired in period t, and countries have identical monetary processes and fixed costs[5]

$$W_t L_t^E = \Pi_t.$$

Thus, the total income received by all workers is equal to total revenues. Defining aggregate employment as $L_t = L_t^P + L_t^E$ this relation can be written

$$W_t L_t = R_t = P_t C_t.$$

Using the labor supply relation in equation (5), one can solve for the aggregate level of employment as a function of aggregate consumption:

$$L_t = \frac{P_t C_t}{\kappa P_t C_t^{\rho}} = \frac{1}{\kappa} C_t^{1-\rho}.$$

C.1.2 Aggregation of Profits Earned by All Firms Operating in the Home Economy

The actual period-t profits of a firm owned by country j and earned in the Home country can be expressed as a function of revenues:

$$\begin{aligned}
\pi_j(\varphi_j(t)) &= p_j(\varphi_j(t))c_j(\varphi_j(t)) - W_t\left(\frac{c_j(\varphi_j(t))}{\varphi_j(t)}\right) - P_t f \\
&= p_j(\varphi_j(t))c_j(\varphi_j(t)) - p_j(\varphi_j(t))c_j(\varphi_j(t))\left(\frac{W_t}{\varphi_j(t)p_j(\varphi_j(t))}\right) - P_t f \\
&= \Gamma_0 r_j(\varphi_j(t)) - P_t f,
\end{aligned}$$

where, substituting the firm's pricing rule from Section 2.3, $\Gamma_0 = 1 - \frac{\alpha\chi E_{t-1}[\frac{1}{M_t}]W_t}{\kappa(1-\beta\theta)}$. The profit of a firm picked at random from the country j-owned sector of the Home economy is then computed using the equilibrium distribution of firm productivity levels:

$$\begin{aligned}
\bar{\pi}_j &= \int_0^\infty \left[\Gamma_0 r_j(\varphi_j(t)) - P_t f\right]\eta_j(\varphi(t))\,d\varphi \\
&= \Gamma_0 \int_0^\infty r_j(\varphi_j(t))\eta_j(\varphi(t))\,d\varphi - P_t f \\
&= \Gamma_0 \int_0^\infty r_j(\bar{\varphi}_{jt})\frac{r_j(\varphi_j(t))}{r_j(\bar{\varphi}_{jt})}\eta_j(\varphi(t))\,d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_{jt}) \int_0^\infty \left(\frac{\varphi_j(t)}{\bar{\varphi}_{jt}}\right)^{\mu-1}\eta_j(\varphi(t))\,d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_{jt}) \left(\frac{1}{\bar{\varphi}_{jt}}\right)^{\mu-1} \int_0^\infty \varphi_j(t)^{\mu-1}\eta_j(\varphi(t))\,d\varphi - P_t f \\
&= \Gamma_0 r_j(\bar{\varphi}_{jt}) - P_t f.
\end{aligned}$$

[4] Alternatively, one could specify that entrepreneurial workers receive a wage that is larger than the manufacturing wage by some constant factor without substantively affecting the results.

[5] The assumption that countries are identical implies that $n_F \pi_F(\bar{\varphi}_F(t)) = n_H^* S_t \pi_H^*(\bar{\varphi}_H(t))$.

Substituting back in the definition of Γ_0 and rearranging, it is clear that the average level of profit is also the level of profit earned by a firm with the average productivity level, or $\bar{\pi}_j = \pi_j(\bar{\varphi}_j(t))$. All firms in each sector can now be treated as though they earned this average level of profit. Total profits earned by all Home- and Foreign-owned firms in the Home economy are then computed as

$$\begin{aligned}\Pi_t &= \int_0^{n_H} \pi_H(\bar{\varphi}_H(t))di + \int_1^{1+n_F} \pi_F(\bar{\varphi}_F(t))di \\ &= n_H \pi_H(\bar{\varphi}_H(t)) + n_F \pi_F(\bar{\varphi}_F(t)).\end{aligned}$$

C.2 Expressing Expected Discounted Profits in Terms of Revenues

As explained in the text, since each Home firm has a different productivity draw, $\varphi_H(t)$, firm subscripts are omitted in the following derivations. Expected discounted profits for the Home firm earned in the Home market in period t can be expressed in terms of revenues:

$$\begin{aligned}E_{t-1}[d_{t-1}\pi_H(\varphi_H(t))] &= E_{t-1}[d_{t-1}(p_H(\varphi_H(t))c_H(\varphi_H(t)) - W_t l_H(\varphi_H(t)) - P_t f)] \\ &= E_{t-1}[d_{t-1}(p_H(\varphi_H(t))c_H(\varphi_H(t)) - \frac{W_t c_H(\varphi_H(t))}{\varphi_H(t)} - f)] \\ &= E_{t-1}[d_{t-1}(p_H(\varphi_H(t))c_H(\varphi_H(t)) \\ &\quad -\frac{\kappa P_t C_t^{\rho} c_H(\varphi_H(t))}{\varphi_H(\varphi_H(t))}\left(\frac{p_H(\varphi_H(t))}{p_H(\varphi_H(t))}\right) - f)] \qquad (1) \\ &= E_{t-1}[d_{t-1}(p_H(\varphi_H(t))c_H(\varphi_H(t)) - \frac{\kappa P_t C_t^{\rho} r_H(\varphi_H(t))}{\varphi_H(t) p_H(\varphi_H(t))} - f)] \\ &= E_{t-1}[d_{t-1}(p_H(\varphi_H(t))c_H(\varphi_H(t)) \\ &\quad -\frac{\kappa P_t \left(\frac{M_t}{P_t}\left(\frac{1-\beta m}{\chi}\right)\right) r_H(\varphi_H(t))}{\varphi_H(t)\left(\frac{\kappa(1-\beta m)}{\alpha\chi\varphi_H(t)E_{t-1}\left[\frac{1}{M_t}\right]}\right)} - f)] \qquad (2) \\ &= E_{t-1}[d_{t-1}(r_H(\varphi_H(t)) - \alpha E\left[\frac{1}{M_t}\right] M_t r_H(\varphi_H(t)) - f)] \\ &= E_{t-1}[d_{t-1}(1 - \alpha E\left[\frac{1}{M_t}\right] M_t) r_H(\varphi_H(t))] - f E_{t-1}[d_{t-1}].\end{aligned}$$

C.3 Reducing the Zero-Cutoff Profit Conditions

The Zero-Cutoff Profit Condition (ZCP) implies that

$$E_{t-1}[d_{t-1}\pi_H(\hat{\varphi}(t))] = E_{t-1}[d_{t-1}(1 - \alpha E\left[\frac{1}{M_t}\right] M_t) r_H(\hat{\varphi}(t))] - P_t f E_{t-1}[d_{t-1}] \equiv 0.$$

Thus, one can write

$$E_{t-1}[d_{t-1}(1 - \alpha E\left[\frac{1}{M_t}\right] M_t) r_H(\hat{\varphi}(t))] = P_t f E_{t-1}[d_{t-1}].$$

Noting from Appendix A that $p_H(\varphi_H(t))c_H(\varphi_H(t)) = p_H(\varphi_H(t))^{1-\mu}P_t^{\mu}C_t = r_H(\varphi_H(t))$ and $d_{t-1} = E_{t-1}\left[\frac{\beta P_{t-1}C_{t-1}^{\rho}}{P_t C_t^{\rho}}\right]$,

$$\begin{aligned}
E_{t-1}[d_{t-1}(1-\alpha E\left[\frac{1}{M_t}\right]M_t)p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\mu}C_t] &= P_t f E_{t-1}[d_{t-1}] \\
E_{t-1}\left[\frac{\beta P_{t-1}C_{t-1}^{\rho}}{P_t C_t^{\rho}}(1-\alpha E\left[\frac{1}{M_t}\right]M_t)p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\mu-1}C_t\right] &= fE_{t-1}\left[\frac{\beta P_{t-1}C_{t-1}^{\rho}}{P_t C_t^{\rho}}\right] \\
\beta P_{t-1}C_{t-1}^{\rho}E_{t-1}\left[\frac{1}{P_t C_t^{\rho}}(1-\alpha E\left[\frac{1}{M_t}\right]M_t)p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\mu-1}C_t\right] &= \beta P_{t-1}C_{t-1}^{\rho}fE_{t-1}\left[\frac{1}{P_t C_t^{\rho}}\right] \\
E_{t-1}\left[\frac{1}{P_t C_t^{\rho}}(1-\alpha E\left[\frac{1}{M_t}\right]M_t)p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\mu-1}C_t\right] &= fE_{t-1}\left[\frac{1}{P_t C_t^{\rho}}\right].
\end{aligned}$$

Using equation (b.13) to substitute for consumption, one finds

$$E_{t-1}\left[\frac{1}{P_t\left(\frac{M_t}{P_t}\left(\frac{1-\beta\theta}{\chi}\right)\right)}(1-\alpha E\left[\frac{1}{M_t}\right]M_t)p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\mu-1}\left(\frac{M_t}{P_t}\left(\frac{1-\beta\theta}{\chi}\right)\right)^{\frac{1}{\rho}}\right] = fE_{t-1}\left[\frac{1}{P_t\left(\frac{M_t}{P_t}\left(\frac{1-\beta\theta}{\chi}\right)\right)}\right]$$

$$\left(\frac{1-\beta\theta}{\chi}\right)^{\frac{1}{\rho}}p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\left(\mu-1-\frac{1}{\rho}\right)}E_{t-1}\left[\left(\frac{1}{M_t}\right)(1-\alpha E\left[\frac{1}{M_t}\right]M_t)M_t^{\frac{1}{\rho}}\right] = fE_{t-1}\left[\frac{1}{M_t}\right]$$

$$\left(\frac{1-\beta\theta}{\chi}\right)^{\frac{1}{\rho}}p_H(\hat{\varphi}(t))^{1-\mu}P_t^{\left(\mu-1-\frac{1}{\rho}\right)}E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}(1-\alpha E\left[\frac{1}{M_t}\right]M_t)\right] = fE_{t-1}\left[\frac{1}{M_t}\right]$$

Substituting the definitions for $p_H(\hat{\varphi}(t))$ and P_t given in equations (15) and (22) from the main text, the expression becomes

$$\left(\frac{1-\beta\theta}{\chi}\right)^{\frac{1}{\rho}}\left(\frac{\kappa(1-\beta\theta)}{\alpha\chi E_{t-1}\left[\frac{1}{M_t}\right]\hat{\varphi}_H(t)}\right)^{1-\mu}\left(\frac{\kappa(1-\beta\theta)N^{\frac{1}{1-\mu}}}{\alpha\chi E\left[\frac{1}{M_t}\right]\bar{\varphi}_t}\right)^{\left(\mu-1-\frac{1}{\rho}\right)} * E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}(1-\alpha E\left[\frac{1}{M_t}\right]M_t)\right] = fE_{t-1}\left[\frac{1}{M_t}\right].$$

Simplifying, one finds

$$N^{\frac{1-\rho(\mu-1)}{\rho(\mu-1)}}\left(\frac{1}{\kappa}\alpha\right)^{\frac{1}{\rho}}\hat{\varphi}_H^{\mu-1}(t)\bar{\varphi}_t^{\frac{1-\rho(\mu-1)}{\rho}}E_{t-1}\left[M_t^{\frac{1-\rho}{\rho}}\left(1-\alpha E_{t-1}\left[\frac{1}{M_t}\right]M_t\right)\right] = fE_{t-1}\left[\frac{1}{M_t}\right]^{\frac{\rho-1}{\rho}}.$$

It is now possible to isolate $\hat{\varphi}_H(t)$ and obtain equation (27) in the main text:

$$\hat{\varphi}_H(t) = \left(\frac{fE_{t-1}[\frac{M_{t-1}}{M_t}]}{a_1 M_{t-1} E_{t-1}\left[\left(\frac{1}{M_t}\right)^{\frac{1-\rho}{\rho}} \left(1 - \alpha E_{t-1}\left[\frac{1}{M_t}\right] M_t\right)\right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}}, \tag{c.1}$$

where $a_1 = N^{\frac{1-\rho(\mu-1)}{\rho(\mu-1)}} \left(\frac{\alpha E_{t-1}[\frac{1}{M_t}]}{\kappa}\right)^{\frac{1}{\rho}}$.

The same process using the ZCP for Foreign firms operating in the Home country yields equation (28),

$$\hat{\varphi}_F(t) = \left(\frac{f_{MNE}E_{t-1}[\frac{M^*_{t-1}}{M^*_t}]}{a_1 M_{t-1} E_{t-1}\left[\left(\frac{1}{M_t}\right)^{\frac{1-\rho}{\rho}} \left(1 - \alpha E_{t-1}\left[\frac{1}{M_t}\right] M_t\right)\right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}}. \tag{c.2}$$

C.4 The Existence of $\hat{\varphi}_H(t)$

Let Z be defined by

$$Z = \hat{\varphi}_H(t) - \left(\frac{fE_{t-1}[\frac{M_{t-1}}{M_t}]}{a_1 M_{t-1} E_{t-1}\left[\left(\frac{1}{M_t}\right)^{\frac{\rho-1}{\rho}} \left(1 - \alpha E_{t-1}\left[\frac{1}{M_t}\right] M_t\right)\right]} \right)^{\frac{1}{\mu-1}} \bar{\varphi}_t^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)}}.$$

Using the definition of $\bar{\varphi}_j(t)$ given in Section 2.5 and recalling that $n_j = 1 - G(\hat{\varphi}_j(t))$, Z can be rewritten:

$$= \hat{\varphi}_H(t) - \Gamma_1 \left(\int_{\hat{\varphi}_H(t)}^{\infty} \varphi_H^{\mu-1}(t) g(\varphi) d\varphi + \int_{\frac{1}{\gamma}\hat{\varphi}_H(t)}^{\infty} \varphi_F^{\mu-1}(t) g(\varphi) d\varphi \right)^{\frac{\rho(\mu-1)-1}{\rho(\mu-1)^2}},$$

where $\Gamma_1 = \left(\frac{fE_{t-1}[\frac{M_{t-1}}{M_t}]}{\left(\frac{\alpha E[\frac{1}{M_t}]}{\kappa}\right)^{\frac{1}{\rho}} M_{t-1}E_{t-1}\left[\left(\frac{1}{M_t}\right)^{\frac{\rho-1}{\rho}}\left(1-\alpha E_{t-1}\left[\frac{1}{M_t}\right]M_t\right)\right]} \right)^{\frac{1}{\mu-1}}$. The partial derivative of Z with respect to $\hat{\varphi}_H(t)$ is

$$\frac{\partial Z}{\partial \hat{\varphi}_H(t)} = 1 - \Gamma_1 \left(\frac{\rho(\mu-1)-1}{\rho(\mu-1)} \right) * \left(\int_{\hat{\varphi}_H(t)}^{\infty} \varphi_H^{\mu-1}(t) g(\varphi) d\varphi + \int_{\frac{1}{\gamma}\hat{\varphi}_H(t)}^{\infty} \varphi_F^{\mu-1}(t) g(\varphi) d\varphi \right)^{\frac{\rho(\mu-1)-1-\rho(\mu-1)^2}{\rho(\mu-1)^2}} * \left(-\hat{\varphi}_H(t) g(\hat{\varphi}_H(t)) - \frac{1}{\gamma}\hat{\varphi}_H(t) g(\hat{\varphi}_H(t))\right),$$

or, rearranging,

$$\frac{\partial Z}{\partial \hat{\varphi}_H(t)} = 1 + \Gamma_1 \left(\frac{\rho(\mu-1)-1}{\rho(\mu-1)} \right) *$$

$$\left(\int_{\hat{\varphi}_H(t)}^{\infty} \varphi_H^{\mu-1}(t) g(\varphi) d\varphi + \int_{\frac{1}{\gamma}\hat{\varphi}_H(t)}^{\infty} \varphi_F^{\mu-1}(t) g(\varphi) d\varphi \right)^{\frac{\rho(\mu-1)-1-\rho(\mu-1)^2}{\rho(\mu-1)^2}} *$$

$$\left(\frac{\gamma+1}{\gamma} \right) \hat{\varphi}_H(t) g(\hat{\varphi}_H(t)).$$

The derivative implies that Z is monotonically increasing as long as Γ_1 is a real number and has the same sign as $\frac{\rho(\mu-1)-1}{\rho(\mu-1)}$. This will be the case if both Γ_1 and $\frac{\rho(\mu-1)-1}{\rho(\mu-1)}$ are greater than zero, which will be the case if (1) $E_{t-1}\left[\left(\frac{1}{M_t} \right)^{\frac{\rho-1}{\rho}} \left(1 - \alpha E_{t-1}\left[\frac{1}{M_t} \right] M_t \right) \right] > 0$ and (2) $\rho(\mu-1)-1 > 0$. Given the process defining the growth rate of the money supply defined in Section 2.2 of the main text, the first condition implies that the markup $(\frac{1}{\alpha})$ is greater than $e^{\frac{1}{\rho}\sigma_m^2}$, a risk-weighted measure of the variance of the money supply growth rate. The condition reduces to $\mu < \frac{e^{\frac{1}{\rho}\sigma_m^2}}{e^{\frac{1}{\rho}\sigma_m^2}-1}$ and holds for a range of estimated values of μ ($1 < \mu \leq 11$, as estimated in Bergin (2003) and reported in Devereux and Lane (2003)) and for variances between 0 and 1 and estimated values of ρ ($1 < \rho < 6$, as reported in Deaton (1992, p.73)), although each value of ρ may satisfy this condition only for a subset of the range $1 < \mu \leq 11$ as σ_m^2 approaches 1. The second condition implies that $\mu > \frac{1}{\rho} + 1$ which, based on previous empirical estimates of μ, is a mild or nonbinding restriction for reasonable values of ρ.

C.5 Calibration

The simulation results are obtained using an exponential distribution for $g(\varphi)$ both because it is numerically tractible when integrating to calculate the aggregate productivity level and because it is attractive in its generality, since it represents a special case of both the gamma and beta distributions. The parameters used are as follows

ρ	2
μ	5
κ	2
χ	1
β	0.96
ψ	.25
ψ^*	.25
f, f_{MNE}	0.1

The value for ρ, the coefficient of relative risk aversion, is taken from Devereux and Lane (2003) and is consistent with estimates reported in Deaton (1992). The coefficient on labor in the utility function, κ, is also the same as in Devereux and Lane (2003). The subjective discount rate in the utility function, β, is assigned the value used in Bergin (2003) and the value for the substitution elasticity between goods is taken from the middle of the range of estimates in that study. To highlight the effect of changes in monetary volatility on the value of γ, it is assumed in the simulations that the mean money supply growth rates are equal across countries ($\bar{m} = \bar{m}^*$) and that there is no extra overhead cost involved in doing business abroad ($f = f_{MNE}$). The values for $\bar{m}$ and $\bar{m}^*$ are set equal to focus on the role of shocks on entry behavior, as well as to satisfy the restriction that $\theta < \frac{1}{\beta}$ for $0 < \sigma_m^2 \leq 1$.

Appendix D: The Case of Incomplete Markets– A Static Example

D.1 Consumer Behavior

In this one-period version of the model without bonds, the representative consumer in the Home country maximizes lifetime utility subject to an income and cash-in-advance (CIA) constraint:

$$\max U(C, L)$$

$$s.t.\ PC = wL + \Pi = M \tag{d.1}$$

where U is a function of aggregate consumption, C, and labor, L,

$$U = \frac{1}{1-\rho} C^{1-\rho} - \kappa L.$$

Aggregate consumption is still an index reflecting preferences with a constant elasticity of substitution (CES) across the set of all available goods,

$$C = \left[\int_0^{n_H} c_H(i)^{\frac{\mu-1}{\mu}} di + \int_1^{1+n_F} c_F(i)^{\frac{\mu-1}{\mu}} di \right]^{\frac{\mu}{\mu-1}}.$$

First order conditions yield the same wage relation,

$$w = \kappa P C^{\rho}. \tag{d.2}$$

The aggregate price index and demand equations are unchanged:

$$P = \left(\int_0^{n_H} p_H(i)^{1-\mu} di + \int_1^{1+n_F} p_F(i)^{1-\mu} di \right)^{\frac{1}{1-\mu}} \tag{d.3}$$

$$c_H(i) = \left(\frac{p_H(i)}{P} \right)^{-\mu} C \qquad c_F(i) = \left(\frac{p_F(i)}{P} \right)^{-\mu} C \tag{d.4}$$

Multiplying by $p_H(i)$, the expression for total expenditure on a particular Home good in the Home country, $r_H(i)$, is derived

$$r_H(i) = \left(\frac{p_H(i)}{P} \right)^{1-\mu} R,$$

where R is total consumer expenditure in the Home country. Note that in the presence of a cash-in-advance constraint, total expenditure equals the money supply, ($M = PC = R$) so that

$$r_H(i) = \left(\frac{p_H(i)}{P}\right)^{1-\mu} M, \tag{d.5}$$

or $r_H(i) = Mp_H(i)^{1-\mu}P^{\mu-1}$.

D.2 Firm Behavior

Firms each produce a unique good and a have a different productivity index, φ, drawn each period as an independent, identically distributed random variable at the point in time the firm decides to enter the industry. Production is linear in labor and is characterized by the technology

$$c_H(i) = \varphi_H(i)l_H(i), \tag{d.6}$$

where $\varphi_H(i)$ is the productivity draw of a Home firm i and $l_H(i)$ is the quantity of labor used by this firm in its domestic plant. Variables for consumption and production activity in the Foreign country are denoted by an asterisk, so that the identical technology for production by a Home firm abroad is represented by

$$c^*_H(i) = \varphi_H(i)l^*_H(i).$$

The firm seeks to maximize the expected market value of total nominal profits from domestic and overseas plants. Producers anticipate potential fluctuations in demand and wages in the host country as a result of volatility in the host-country money supply. In addition, they consider potential fluctuations in the exchange rate, which could occur due to shocks to either the host-country or its own money supply (that is, to M or M^*), when deciding whether to enter the overseas market. Firms put a subjective value on each potential state of the economy using a stochastic discount factor, U_c, for Home firms and U_{c^*} for Foreign firms. Marginal utility is used as a discount factor because it measures how much a shock will impact the well-being of the consumers who own the firm (Cochrane 2001, Chapter 1). Thus, the firm's problem is to decide whether and how much to produce in its own country or in both countries:

$$\max E[U_c\pi^T_H(i)],$$

where

$$\pi^T_H(i) = E[U_c\pi_H(i)] + \max\{0, E[U_c\pi^*_H(i)]\},$$

$$\pi_H(i) = p_H(i)c_H(i) - wl_H(i) - f, \tag{d.7}$$

and

$$\pi^*_H(i) = Sp^*_H(i)c^*_H(i) - Sw^*l^*_H(i) - f^*_{MNE}. \tag{d.8}$$

Each Home company pays a fixed overhead cost, f, in the Home country and f^*_{MNE} in the Foreign country (if it invests overseas to become a multinational enterprise) that must be subtracted to determine profit. Labor is hired and fixed *overhead* costs are paid after the firm finds out its unique productivity draw, $\varphi_H(i)$. Therefore, to calculate profits from operations abroad, a Home firm takes into account the exchange rate, S, at which it will have to pay wages for Foreign workers and repatriate revenues earned overseas, as well as the fixed overhead costs. It is useful to emphasize here that this is a model of *horizontal* direct investment, where a firm produces a unique good in multiple countries, but for local consumption in each country. The model abstracts from cross-border flows of physical capital.[6] FDI in the case here is the payment of some fixed cost (f^*_{MNE}, denominated in the currency of the payee– in this case, the Home firm) to gain entry into the local market for a particular period.

Substituting $\frac{c_H(i)}{\varphi_H(i)}$ for $l_H(i)$ and taking the derivative of $E[U_c \pi^T_H(i)]$ with respect to $c_H(i)$, one can determine the firm's pricing behavior:

$$\frac{\partial E[U_c \pi^T_H(i)]}{\partial c(i)} : E\left[U_c\left(p_H(i) + \left(\frac{\partial p_H(i)}{\partial c_H(i)}\right) c_H(i) - \frac{w}{\varphi_H(i)}\right)\right] = 0$$

$$E\left[C^{-\rho}\left(p_H(i) + \left(-\frac{1}{\mu}\right) p_H(i) - \frac{w}{\varphi_H(i)}\right)\right] = 0$$

$$p_H(i) = \left(\frac{\mu}{\mu - 1}\right) \frac{E\left[C^{-\rho}\left(\frac{w}{\varphi_H(i)}\right)\right]}{E\left[C^{-\rho}\right]}.$$

A firm will set a price for its unique good equal to a fixed markup over the expected discounted marginal cost. Defining $\alpha = \frac{\mu-1}{\mu}$– the inverse of the markup– and noting that the firm knows its own productivity level at the time it makes its production decisions, the Home firm's pricing rule for its good in the domestic market can be written as

$$p_H(\varphi_H) = \frac{E\left[U_c w\right]}{\alpha \varphi_H E\left[U_c\right]}.$$

An analogous equation applies for the firm's pricing in the Foreign market. Expressed in terms of Foreign currency, the Home firm's price abroad is

$$p^*_H(\varphi_H) = \frac{E\left[U_c S w^*\right]}{\alpha \varphi_H E\left[U_c S\right]}$$

For foreign firms, the pricing rules are

$$\begin{aligned} p_F(\varphi_F) &= \frac{E\left[U_{c*}\left(\frac{w}{S}\right)\right]}{\alpha \varphi_F E\left[U_{c*}\left(\frac{1}{S}\right)\right]} \\ p^*_F(\varphi_F) &= \frac{E\left[U_{c*} w^*\right]}{\alpha \varphi_F E\left[U_{c*}\right]} \end{aligned}$$

[6] Cross-border capital flows, where overseas capital is used in domestic production using domestic technology, are distinct from FDI, where firms use a common technology to produce identical goods in multiple countries. See Russ (2002) for a more detailed discussion and a literature review of studies modeling cross-border capital flows.

Substituting the CIA constraint and the wage relation, equation (d.2), the pricing rules in (d.9) become

$$p_H(\varphi_H) = \frac{\kappa P^{1-\rho}}{\alpha \varphi_H E\left[M^{-\rho}\right]} \tag{d.9a}$$

$$p_H^*(\varphi_H) = \frac{\kappa P^{*1-\rho} E\left[M^{-\rho} S M^{*\rho}\right]}{\alpha \varphi_H E\left[M^{-\rho} S\right]} \tag{d.9b}$$

$$p_F(\varphi_F) = \frac{\kappa P^{1-\rho} E\left[M^{*-\rho}\left(\frac{1}{S}\right) M^{\rho}\right]}{\alpha \varphi_F E\left[M^{*-\rho}\left(\frac{1}{S}\right)\right]} \tag{d.9c}$$

$$p_F^*(\varphi_F) = \frac{\kappa P^{*1-\rho}}{\alpha \varphi_F E\left[M^{*-\rho}\right]} \tag{d.9d}$$

D.3 Aggregation

In equilibrium, there will be a continuum of goods produced by Home firms over $[0, n_H]$ and by Foreign firms over $(1, n_F]$ $(n_H, n_F \leq 1)$in the Home market. There is also an equilibrium distribution of productivity levels– $\eta_H(\varphi)$ for Home firms and $\eta_F(\varphi)$ for Foreign firms, each with positive support over a subset of $(0, \infty)$. As in Section 2.4 of the main text, the aggregate price index is computed as

$$P = \left[\int_0^{n_H} \int_0^{\infty} p_H(\varphi_H)^{1-\mu} \eta_H(\varphi)\, d\varphi di + \int_1^{1+n_F} \int_0^{\infty} p_F(\varphi_F)^{1-\mu} \eta_F(\varphi)\, d\varphi di\right]^{\frac{1}{1-\mu}}$$

The pricing rules, (d.9a) and (d.9b); the wage relation (d.2); and the CIA constraint can be used to reduce this expression to a function of the number of firms and the average per-unit labor cost. That is,

$$P = \left[\begin{array}{c} \int_0^{n_H} \int_0^{\infty} \left(\frac{\kappa P^{1-\rho}}{\alpha \varphi_H E[M^{-\rho}]}\right)^{1-\mu} \eta_H(\varphi)\, d\varphi di + \\ \int_1^{1+n_F} \int_0^{\infty} \left(\frac{\kappa P^{1-\rho} E\left[M^{*-\rho}\left(\frac{1}{S}\right) M^{\rho}\right]}{\alpha \varphi_F E\left[M^{*-\rho}\left(\frac{1}{S}\right)\right]}\right)^{1-\mu} \eta_F(\varphi)\, d\varphi di \end{array}\right]^{\frac{1}{1-\mu}}.$$

The distribution of productivity levels from which firms draw is the same for each firm (for all i), so that the expression can be simplified:

$$\begin{aligned} P &= \frac{P^{1-\rho}}{\alpha}\left[n_H b_1 \int_0^{\infty} \varphi_H{}^{\mu-1} \eta_H(\varphi)\, d\varphi + n_F b_2 \int_0^{\infty} \varphi_F{}^{\mu-1} \eta_F(\varphi)\, d\varphi\right]^{\frac{1}{1-\mu}} \\ &= \left(\frac{1}{\alpha}\right)^{\frac{1}{\rho}}\left[n_H b_1 \int_0^{\infty} \varphi_H{}^{\mu-1} \eta_H(\varphi)\, d\varphi + n_F b_2 \int_0^{\infty} \varphi_F{}^{\mu-1} \eta_F(\varphi)\, d\varphi\right]^{\frac{1}{\rho(1-\mu)}}, \end{aligned}$$

where $b_1 = \left(\frac{\kappa}{E[M^{-\rho}]}\right)^{1-\mu}$ and $b_2 = \left(\frac{\kappa E\left[M^{*-\rho}\left(\frac{1}{S}\right)M^{\rho}\right]}{E\left[M^{*-\rho}\left(\frac{1}{S}\right)\right]}\right)^{1-\mu}$. Let $\bar{\varphi}_H$ and $\bar{\varphi}$, defined by

$$\begin{aligned}
\bar{\varphi}_H &= \left(\int_0^{\infty} {\varphi_H}^{\mu-1} \eta_H(\varphi)\, d\varphi\right)^{\frac{1}{\mu-1}} \\
\bar{\varphi}_F &= \left(\int_0^{\infty} {\varphi_F}^{\mu-1} \eta_H(\varphi)\, d\varphi\right)^{\frac{1}{\mu-1}},
\end{aligned}$$

be the average level of productivity of Home and Foreign firms, respectively, operating in the Home economy.[7] Then, the average per-unit labor cost for the entire Home economy is

$$\bar{\varphi} = \left[\frac{n_H b_1}{N}\bar{\varphi}_H + \frac{n_F b_2}{N}\bar{\varphi}_H\right]^{\frac{1}{\mu-1}}, \tag{d.10}$$

This average per-unit labor cost, $\bar{\varphi}$, is the aggregate level of productivity in the Home economy weighted by Home and Foreign-owned firms' expectations of the subjectively discounted nominal wage. It is useful to note that the weights b_1 and b_2 would be equal if the monetary processes of both countries were equal and the exchange rate, S, was fixed at some level, $\bar{S}$ for instance.

Using (d.10), the aggregate price level can now be expressed as

$$P = \left(\frac{N^{\frac{1}{1-\mu}}\kappa}{\alpha\bar{\varphi}}\right)^{\frac{1}{\rho}}. \tag{d.11}$$

D.4 Equilibrium Conditions

D.4.1 The Cutoff Productivity Level

Given the distribution of productivity levels, there will be some Home firm with productivity level $\hat{\varphi}_H$ such that profits equal zero. Any producer with a productivity level below $\hat{\varphi}_H$ will immediately exit the market, knowing that it will realize negative profits should it remain in the industry. Hence, a zero-cutoff profit condition (ZCP) can be defined for the Home firm operating in the Home and Foreign markets,

$$E[U_c \pi_H(\hat{\varphi}_H)] = 0 \tag{d.12a}$$
$$E[U_c \pi_H^*(\hat{\varphi}_H^*)] = 0, \tag{d.12b}$$

and the Foreign firm in the Foreign in the Home and Foreign markets,

$$E[U_{c^*} \pi_F(\hat{\varphi}_F)] = 0 \tag{d.12c}$$
$$E[U_{c^*} \pi_F^*(\hat{\varphi}_F^*)] = 0 \tag{d.12d}$$

where $\hat{\varphi}_j^*$ ($j \in \{H, F\}$) represents the cutoff level for the Home- and Foreign-owned firm in the Foreign market. Using the ZCP condition, one can derive the profit level of the average Home firm in the domestic market as a function of $\hat{\varphi}_H$ and f.

[7] In actuality, $\bar{\varphi}_H$ and $\bar{\varphi}_F$, are expressions of the production-weighted harmonic mean of productivity levels for Home and Foreign firms operating in the Home economy.

First, it is useful to express $E[U_c\pi_H(\varphi_H)]$ and $E[U_{c^*}\pi_F(\varphi_F)]$ as functions of revenue. Beginning with the definition of domestic profits from equation (d.7) and making the appropriate substitutions using the CIA constraint and the wage relation (expressions (d.1) and (d.2))

$$\begin{aligned}
E[U_c\pi_H(\varphi_H)] &= E[U_c(p_H(\varphi_H)c_H(\varphi_H) - wl_H(\varphi_H) - f)] \\
&= E[C^{-\rho}(p_H(\varphi_H)c_H(\varphi_H) - \frac{wc_H(\varphi_H)}{\varphi_H(\varphi_H)} - f)] \\
&= E[C^{-\rho}(p_H(\varphi_H)c_H(\varphi_H) - \alpha M^\rho E\left[M^{-\rho}\right] p_H(\varphi_H)c_H(\varphi_H) - f)] \\
&= E[C^{-\rho}(1 - \alpha M^\rho E\left[M^{-\rho}\right])p_H(\varphi_H)c_H(\varphi_H)] - fE[C^{-\rho}]
\end{aligned}$$

Calling firm revenues $r_H(\varphi_H)$, expected discounted profits for Home firm i in its native market can be written

$$E[U_c\pi_H(\varphi_H)] = E[C^{-\rho}(1 - \alpha M^\rho E\left[M^{-\rho}\right])r_H(\varphi_H)] - fE[C^{-\rho}].$$

Substituting the expressions for revenue (d.5), the aggregate price level (d.11), and the pricing rule (d.9a), a firm's expected discounted profits can be written in terms of the domestic money supply, the markup, and its relative productivity level,

$$E[U_c\pi_H(\varphi_H)] = \left(\frac{\varphi_H}{\bar{\varphi}}\right)^{\mu-1}\left(\frac{b_1}{N}\right)E[M^{1-\rho}(1 - \alpha M^\rho E\left[M^{-\rho}\right])] - fE[M^{-\rho}]. \quad \text{(d.13)}$$

Similarly, for a Foreign firm investing in the home country as a multinational enterprise, expected discounted profits from operations in the Home country (expressed in terms of the Foreign-country currency) are represented by

$$E[U_{c^*}\pi_F(\varphi_F)] = \left(\frac{\varphi_F}{\bar{\varphi}}\right)^{\mu-1}\left(\frac{b_2}{N}\right)E[M^{*-\rho}\left(\frac{1}{S}\right)M(1 - \alpha M^\rho E\left[M^{-\rho}\right])] - f_{MNE}E[M^{*-\rho}]. \quad \text{(d.14)}$$

The equations for expected discounted profits are very similar for Home- and Foreign-owned firms. The principle differences lie in three important places: (1) the respective discount factors, which are rooted in the monetary conditions expected to emerge in each firm owner's *native* country;[8] (2) the weights for the expected discounted wages (b_1 and b_2), which take into account the impact of fluctuations in the exchange rate on the labor costs faced by the entering Foreign firm, but which do not affect the Home firm operating in its own country; and (3) the explicit introduction of the exchange rate, $\frac{1}{S}$, into the Foreign firm's calculation of expected revenues. One can observe that if the exchange rate were fixed and conditions in both countries were governed by a common monetary innovation or monetary authority, a Home and Foreign firm's expected discounted profits from sales in the Home-country market would be distinguishable only by their unique productivity levels, $\varphi_H(i)$ and φ_F.

Combining the ZCP conditions (d.12a) and (d.12b) with the semi-reduced form equations for expected profits in (d.13) and (d.14), one attains expressions relating the cutoff productivity levels

[8] Explicitly, the discount factor for residents of the Home country is $E[U_c] = E[C^{-\rho}] = E[M^{-\rho}]$, whereas the discount factor for residents of the Foreign country is $E[U_{c^*}] = E[C^{*-\rho}] = E[M^{*-\rho}]$.

for both Home- and Foreign-owned firms to the average per-unit production cost in the Home economy:

$$\left(\frac{\hat{\varphi}_H}{\bar{\varphi}}\right)^{\mu-1}\left(\frac{b_1}{N}\right)E[M^{1-\rho}(1-\alpha M^{\rho}E\left[M^{-\rho}\right])] = fE[M^{-\rho}]$$

$$\hat{\varphi}_H = \left(\frac{fE[M^{-\rho}]}{\left(\frac{b_1}{N}\right)E[M^{1-\rho}(1-\alpha M^{\rho}E\left[M^{-\rho}\right])]}\right)^{\frac{1}{\mu-1}}\bar{\varphi} \tag{d.15}$$

$$\left(\frac{\hat{\varphi}_F}{\bar{\varphi}}\right)^{\mu-1}\left(\frac{b_2}{N}\right)E[M^{*-\rho}\left(\frac{1}{S}\right)M(1-\alpha M^{\rho}E\left[M^{-\rho}\right])] = f_{MNE}E[M^{*-\rho}]$$

$$\hat{\varphi}_F = \left(\frac{f_{MNE}E[M^{*-\rho}]}{\left(\frac{b_2}{N}\right)E[M^{*-\rho}\left(\frac{1}{S}\right)M(1-\alpha M^{\rho}E\left[M^{-\rho}\right])]}\right)^{\frac{1}{\mu-1}}\bar{\varphi} \tag{d.16}$$

It is assumed that the distribution of possible productivity levels for firm i, $g(\varphi)$ is the same for all firms. It is also useful to generalize the notation for the cutoff productivity level: $\hat{\varphi} = \varphi(\hat{\imath}_j)$. A Home firm which draws $\varphi < \hat{\varphi}_H$, for instance, will immediately exit– before initiating production. The distribution of successful firms' productivity levels, $\eta_j(\varphi)$, is therefore the probability of drawing a particular φ, given that $\varphi \geq \hat{\varphi}_j$. Let $G(\varphi)$ be the cumulative distribution of the probability density $g(\varphi)$. Then, as in Melitz (2002), the equilibrium distribution for Home firms is therefore defined by

$$\eta_H(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\hat{\varphi}_H)} \text{ if } \varphi \geq \hat{\varphi}_H \\ 0 \text{ if } \varphi < \hat{\varphi}_H. \end{cases} \tag{d.17}$$

There is a similar distribution for Foreign firms operating in the Home market,

$$\eta_F(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\hat{\varphi}_F)} \text{ if } \varphi \geq \hat{\varphi}_F \\ 0 \text{ if } \varphi < \hat{\varphi}_F^*. \end{cases} \tag{d.18}$$

From (d.17) and (d.18), it is clear that the average per-unit labor cost, equation (d.10), is actually a function of the cutoff productivity levels:

$$\bar{\varphi} = \bar{\varphi}(\hat{\varphi}_H, \hat{\varphi}_F) = \left[\frac{n_H}{N}\bar{\varphi}_H^{\mu-1} + \frac{n_F}{N}\bar{\varphi}_F^{\mu-1}\right]^{\frac{1}{\mu-1}}, \tag{d.19}$$

where

$$\bar{\varphi}_H = \bar{\varphi}(\hat{\varphi}_H) = \left[\frac{1}{1-G(\hat{\varphi}_H)}\int_{\hat{\varphi}_H}^{\infty}\varphi^{\mu-1}g(\varphi)d\varphi\right]^{\frac{1}{\mu-1}} \tag{d.20a}$$

$$\bar{\varphi}_F = \bar{\varphi}(\hat{\varphi}_F) = \left[\frac{1}{1-G(\hat{\varphi}_F)}\int_{\hat{\varphi}_F}^{\infty}\varphi^{\mu-1}g(\varphi)d\varphi\right]^{\frac{1}{\mu-1}} \tag{d.20b}$$

Thus, (d.15) and (d.16) provide two equations with three unknows: the cutoff productivity levels for the Home market, $\hat{\varphi}_H$ and $\hat{\varphi}_F$, and the nominal exchange rate, S. Dividing (d.16) by (d.15) and rearranging, one finds $\hat{\varphi}_F$ as a function of $\hat{\varphi}_H$ and S:

$$\hat{\varphi}_F = \left(\frac{b_1 f_{MNE} E[M^{*-\rho}] E[M^{1-\rho}(1 - \alpha M^{\rho} E[M^{-\rho}])}{b_2 f E[M^{-\rho}] E[M^{*1-\rho}(1 - \alpha M^{\rho} E[M^{-\rho}])} \right)^{\frac{1}{\mu-1}} \hat{\varphi}_H \tag{d.21}$$

Finding a reduced form for the nominal exchange rate will allow the system to be solved and indicate how volatility in the monetary variables underlying the exchange rate impact the willingness of Foreign-owned firms to invest in the Home-country market.

D.4.2 Clearing the Foreign Exchange Market

Thus, to solve the model, it is necessary to solve for the nominal exchange rate. In the case of incomplete markets, this must be done by introducing a market-clearing condition in the market for foreign exchange.[9] First, it is noted that by assumption, all goods consumed by Home-country residents must be produced and purchased within the Home country, in Home-country currency. An analogous rule applies for goods consumed by residents of the Foreign country. Therefore, all revenues earned by Home-country firms from sales in the Foreign market must be expatriated to the Home country, and revenues earned by Foreign-owned firms in the Home market must also be expatriated to the Foreign country. The exchange of revenues from overseas plants is made at an exchange rate, S, which is the equilibrating engine in the market for foreign currency.

The market-clearing condition[10] for foreign currency can therefore be expressed as

$$n_F r(\bar{\varphi}_F) = S n_H^* r^*(\bar{\varphi}_H^*), \tag{d.22}$$

As defined in (d.20a), $\bar{\varphi}_F$ is the average productivity level of Foreign firms in the Home country and $\bar{\varphi}_H^*$ is the same index for Home firms operating abroad. If both countries are identical, with identical processes for monetary shocks, the average level of productivity of overseas plants will be the same for Home- and Foreign-owned firms ($\bar{\varphi}_F = \bar{\varphi}_H^*$), the proportion of prospective Home- and Foreign-owned entrants that choose to invest abroad will be equal ($n_F = n_H^*$), as will the average price multinationals charge for their good in overseas markets. The symmetry of the model [11]

[9] In the intertemporal complete-markets case with money in the utility function introduced in Section 2, the nominal exchange rate is fully specified from the first-order conditions for money and bonds. The for-ex market in that case clears entirely due to consumer's ability to share risk, without a separate market-clearing condition.

[10] The condition here is specified in the spirit of Bacchetta and van Wincoop's (2000) foreign exchange market in their model of exchange rate volatility and trade.

[11] As well as the expression for revenue/expenditure from an individual good defined in equation (d.5).

therefore allows one to find a reduced form for S :

$$\begin{aligned} S &= \frac{p_{MNE} r(\bar{\varphi}_F)}{p^*_{MNE} r^*(\bar{\varphi}^*_H)} \\ &= \frac{r(\bar{\varphi}_F)}{r^*(\bar{\varphi}^*_H)} \\ &= \frac{M(p_F(\bar{\varphi}_F)P^{-1})^{1-\mu}}{M^*(p^*_H(\bar{\varphi}^*_H)P^{*-1})^{1-\mu}} \\ &= \frac{M}{M^*} \end{aligned} \tag{d.23}$$

With a reduced form for S, (d.21) becomes

$$\hat{\varphi}_F = \left(\left(\frac{b_1 f_{MNE} E[M^{*-\rho}] E[M^{1-\rho}(1-\alpha M^{\rho} E[M^{-\rho}])}{b_2 f E[M^{-\rho}] E[M^{*1-\rho}(1-\alpha M^{\rho} E[M^{-\rho}])} \right)^{\frac{1}{\mu-1}} \right) \hat{\varphi}_H, \tag{d.24}$$

which relates the threshold productivity level of Foreign firms operating in the Home economy to that of the Home firms in terms of the fundamental variables, M and M^*; the fixed costs, f and f_{MNE}; the elasticity of substitution, μ (recall $\alpha = \frac{\mu-1}{\mu}$); and the coefficient of relative risk aversion, ρ.

D.5 The Effect of Exchange-Rate Volatility on Firm Entry

The relationship between the threshold productivity levels of Home and Foreign firms operating in the Home market reveals the effect of exchange-rate volatility steon the willingness of Foreign investors to engage in ventures overseas. If $\hat{\varphi}_H$ is greater than $\hat{\varphi}_F$, that means that the least productive Foreign firms will have to be more productive than the least productive Home firms producing in the Home country[12] Thus, one can take the derivative of $\frac{\hat{\varphi}_H}{\hat{\varphi}_F}$ with respect to the underlying parameters to determine how they impact the relative ability of Foreign firms to enter the Home market without expecting to go bankrupt. In addition to permitting equation (d.15) to be solved for $\hat{\varphi}_H$, equation (d.24) provides the expression for the ratio of the productivity level of the least productive Foreign and Home firm which allows one to conduct comparative statics.

To conduct the analysis, it is assumed that the Home and Foreign money supplies are both lognormally distributed. The growth process is defined by

$$m = \bar{m} + \nu,$$

where m is log of the Home money supply and ν is randomly distributed with mean 0 and variance σ^2_m. Let the ratio $\frac{\hat{\varphi}_F}{\hat{\varphi}_H}$ in the setting of incomplete asset markets be called γ_{IM}. Under this money supply process assumed,

$$\frac{\hat{\varphi}_F}{\hat{\varphi}_H} = \left(\frac{f_{MNE} e^{\bar{m}[1-\rho(\mu-1)]-[\rho(\mu-1)-\frac{1}{2}]\sigma^2_m} \left(e^{-\rho\sigma^2_m} - \alpha \right)}{f e^{\left(\frac{1}{2}-\rho\right)\sigma^2_{m^*}} \left(1 - \alpha e^{\rho^2 \sigma^2_m} \right)} \right)^{\frac{1}{\mu-1}} \equiv \gamma_{IM}. \tag{d.25}$$

[12] Since the cumulative distribution of productivity draws, $G(\varphi)$, is monotonically increasing in φ, the probability that a Foreign firm successfully entering the Home market, $p_{ME} = \frac{1-G(\hat{\varphi}_F)}{1-G(\hat{\varphi}^*_F)}$, will fall as $\hat{\varphi}_F$ increases.

Equation (d.25) shows that the behavior of Foreign firms investing in the Home country is impacted by volatility in both the Home and Foreign money supply under a floating exchange rate. As the volatility of the Foreign money supply increases, for typical values of ρ, the change in γ_{IM} is negative[13]

$$\frac{\partial\gamma}{\partial\sigma^2_{M^*}} = \left(\frac{1}{\mu-1}\right)\left(\rho-\frac{1}{2}\right)\gamma_{IM} > 0 \text{ for } \rho \geq 1,$$

implying that increases in the volatility of the exchange rate stemming from fluctuations in the Foreign money supply will increase the cutoff productivity level of Foreign firms relative to that of Home firms entering the Home market, complimenting de Campa's (1993) finding that exchange-rate volatility can deter entry by Foreign firms. Also important is that this deterrent effect increases with the size of local fixed costs (f_{MNE}). Finally, it is evident that the impact of exchange-rate volatility arising from fluctuations in the Home (host-country) money supply is positive with the added restriction that μ exceeds 1.5 (implying a markup with an upperbound of 300%), which fits with previous estimates of μ discussed in Appendices C.4 and C.5.

[13] The derivative is positive as long as the condition ensuring that the threshold productivity level exists still holds. That is, that the markup, $\frac{1}{\alpha}$, is greater than a risk-adjusted measure of the volatility of the Home money supply, $e^{\rho^2\sigma^2}$. For plausible values of ρ ($\rho \geq 1$), this implies that the markup is also greater than $e^{\rho\sigma^2}$.